How Two Key Tenets of Relativity Lead to a Self-Contradiction
By David Johnson

Could a man be younger than himself? I assume you will say no, because that would be self-contradictory, but espousing Einstein's Theory of Relativity would commit you to saying yes.

In the theory there is no absolute state of rest, or absolute motion, and no preferred frame of reference; because of this, one is always entitled to consider himself or herself to be at rest. Yet Einstein also believes that if one is traveling at a high speed relative to light, say half the speed of light, there will be time dilation, or a slowing of time for that reference frame.

Suppose that we have a spaceship that is moving at a steady velocity of .5c and has been doing so for a while. According to Einstein the reference frame and everything in it are experiencing time dilation (meaning that time slows down). But what if an observer in that spaceship considers it to be at rest, which, according to the theory, he is always entitled to do? In that case there would be no time dilation for his reference frame because he is not moving at all. Thus, we get two different, but according to the theory, equally correct answers for how much time has passed for his reference frame depending upon whether he considers himself to be at rest or in motion. If he considers himself to be and to have been in motion he will be younger than if he considers himself to have been at rest. Therefore, according to the theory, he is younger than himself.

It is common in Relativity for observers in different reference frames to perceive the passage of time differently and to have contradictory accounts of how much time has passed. But this is different because this occurs in the same reference frame for the same observer. It is not just two contradictory accounts of time by different observers in separate frames of reference, it is a self-contradiction concerning how much time has passed for that reference frame.

Someone cannot be both 36 and 37 years old at once; something has to give. Either the claim of time dilation is false, the claim that one is always entitled to consider himself or herself to be at rest is false, or they are both false.

But perhaps an apologist for Relativity may attempt to defend it by arguing that the claim is equivalent to a disjunction (an 'or' statement') rather than a conjunction (an 'and' statement). This hypothetical defender may attempt to argue that what Einstein means is that the astronaut could be either 36 years old or 37 years old, not both simultaneously.

I don't know how much this would even help though. Suppose that the trip was long and we adjust the speed so that the difference in age would be greater. If somebody asked the returned astronaut how old he was and he replied 'I'm either 43 or 84 depending on how you look at it' do you really think that makes sense? When he looks in the mirror will he see a 43 year old man or an 84 year old man? Would what he sees as his reflection really depend upon whether he

considered himself to be moving or at rest while on the spaceship? To me this does not work either, but it is important to consider all possibilities, so let's analyze whether one could think of the claim as an 'or' statement.

We need to first make a distinction between two types of disjunction, an 'inclusive or' and an 'exclusive or'. The latter is equivalent to 'either . . . or . . .' meaning that one and only one of the simple statements (the technical term is 'disjunct') is true and the other is false. For example, if you were to say 'It is either raining or it is not' then one disjunct must be true and the other must be false. They are mutually exclusive because they contradict each other so it could not be the case that both are true at the same time. The 'exclusive or' is logically equivalent to saying 'either A or B but not both'.

An 'inclusive or', on the other hand, is a hybrid that comes from combining a conjunction, an 'and' statement, with an 'exclusive or' statement. It is equivalent to saying 'and/or' meaning that it could be either logical operator. For a conjunction, both simple statements (the 'conjuncts') must be true in order for the whole statement to be true. We can combine the truth table for a conjunction with the truth table for the 'exclusive or' to derive the truth table for the 'inclusive or', which gives the result that there are three out of four possible scenarios in which the statement is true: if either disjunct is true and the other is false then the statement as a whole is true, and if both disjuncts are true then the statement as a whole is true. For example, if one were to say 'It is raining or it is Tuesday' then the statement as a whole is false only if neither one of the disjuncts are true. If either one is true then the whole statement is true, and since the two simple statements are not mutually exclusive they can both be true at once; if they are (in other words if it is both raining and a Tuesday) then technically the statement as a whole is true in that case as well, although it might sound a little odd because we do not usually use an 'or' statement that way. To make things more clear the speaker should specify that it is an 'inclusive or', as in: 'It is raining and/or it is Tuesday'.

So the question for this hypothetical apologist is what type of 'or' claim Einstein would be making. I don't think it could be an 'inclusive or' because two different ages would be mutually exclusive. The whole purpose for saying that it was an 'or' statement rather than an 'and' statement in the first place was that it could not be the case that both of them are true at once, which eliminates the possibility that what he meant was equivalent to an 'inclusive or' statement.

It is similar to the claim: 'Today is either Wednesday or not Wednesday'. If interpreted as an 'exclusive or' statement this is a tautology, which means that it is always true. See the following truth table. (Note: The symbol ~ stands for negation or 'not', and the 'v' symbol stands for 'or'):

```
   W   W v ~W
1. T   T  T  FT
2. F   F  T  TF
```

The two disjuncts are contradictory statements, and it must be the case that one or the other of them is true, which means that the statement as a whole is always true. This is demonstrated in the table by the fact that under the main operator it is true on both lines.

But suppose that we chose to interpret this statement as an 'inclusive or' instead. What would be the truth value of it then? An 'exclusive or' is logically equivalent to: (A v ~A) ^ ~ (A ^ ~A). This means 'A or not A, but not both A and not A'. (The symbol ^ stands for 'and'.) This is of course correct for the statement that we are using. To get a truth table that is equivalent to the 'inclusive or' we simply remove the negation that is in front of 'A and not A', like this: (A v ~A) ^ (A ^ ~A). This is translated as: 'A or not A, and A and not A.' I said earlier that an 'inclusive or' statement was like a hybrid between a conjunction and an 'exclusive or' disjunction, and we see that here, as we are simply conjoining the two statements. Let's look at what happens when we use this form of 'or' for 'Today is either Wednesday or not Wednesday'.

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   W   (W v ~W) ^ (W ^ ~W)
1. T    T T FT  F  T F FT
2. F    F T TF  F  F F TF
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There are false truth values under the main operator (center) for both lines of the truth table. This shows that the statement is self-contradictory. So the claim is tautological if considered an 'exclusive or', but self-contradictory if considered an 'inclusive or'. Here is the truth table for the 'exclusive or' for this claim once again, and what it is logically equivalent to, as stated above:

```
   W   (W v ~W)                W    (W v ~W) ^  ~ (W ^ ~W)
1. T    T T FT                 T    T T FT  T  T T F FT
2. F    F T TF                 F    F T TF  T  T F F TF
```

If Einstein's claim is considered an 'exclusive or' that would avoid the self-contradiction, but would it be an accurate reflection of what he actually said? That would make the claim tautological, and while a tautology is always true ('either the reference frame was moving or it was not'), it does not tell us anything new. It seems clear that Einstein intended to go beyond merely asserting something that is self-evident. Also, I do not believe that this would be the correct interpretation because that would mean that one of the two disjuncts is false. That is something that Einstein specifically rejected. He did not believe that one was true and the other false, he thought that both were equally correct descriptions of the relative motion. That is equivalent to asserting that both are true, which at best (for Einstein) would mean that it has to be considered an 'inclusive or'. But an 'inclusive or' would be self-contradictory if the two disjuncts are mutually exclusive, as demonstrated above with 'Today is either Wednesday or not Wednesday', and it obviously is mutually exclusive in this case: either the reference frame was moving and there was time dilation or it was not moving and there was no time dilation. (It would make no sense to say 'The reference frame is moving and/or not moving' as though both could be true at once.) And, really, a far more accurate translation of what he actually said would be a conjunction.

A conjunction of two mutually exclusive claims results in a self-contradiction for the same reason it does with an 'inclusive or': the conjuncts cannot really both be true at once, yet the conjunction asserts that they are. It is like asserting both A and not A at once, or something similar to that. If the astronaut is 36 years old then it is implied that he is not 37 years old, or any other age as measured in years. If he is 37 then he is not 36. One could derive this from natural deduction.

In the following proofs 'A' stands for 'the astronaut is 36 years old' and B stands for 'the astronaut is 37 years old (or some age other than 36, as measured in years)'. (Note: The arrow symbol $\rightarrow$ stands for 'if . . . then', or a conditional.)

Scenario 1: The reference frame was moving   Scenario 2: The reference frame was at rest

1. A			1. B	
2. A $\rightarrow$ ~B			2. B $\rightarrow$ ~A	
3. ~B	1, 2, Modus ponens		3. ~A	1, 2, Modus ponens
4. A ^ ~B	1, 3, Conjunction		4. B ^ ~A	1, 3, Conjunction

So the claim would be that he is '36 and not 37' if he was moving and '37 and not 36' if he was at rest, or (A ^ ~B) ^ (B ^ ~A). Since Einstein is asserting that both of these are equally correct ways of describing relative motion it is like he is saying that both of them are true at once. But this claim is self-contradictory. Here is the truth table:

```
   A B   (A ^ ~B) ^ (B ^ ~A)
1. T T   T F FT  F  T F  FT
2. T F   T T TF  F  F F  FT
3. F T   F F FT  F  T T  TF
4. F F   F F TF  F  F F  TF
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Under the main operator (in the middle) the truth value that is derived is false on every line of the table. This means that for every possible scenario there is none in which the claim is true. Thus, it is self-contradictory. It is asserting A and ~A as well as B and ~B. Another way in which the claim is self-contradictory is that Einstein is asserting both that the reference frame is moving and also that it is not moving, which is the reason that he is committed to saying that there both is and is not time dilation for that reference frame.

Some may wish to resist this outcome, thinking that surely Einstein could not have made such an obvious mistake since he is considered to be perhaps the greatest genius of all time, and they believe that Relativity is true. But he probably just didn't realize the full logical implications of the various claims that he was making. That can happen to anybody. It only seems obvious once it has been pointed out.