An Alternative Version of Categorical Logic

by David Johnson

The 'Some' Problem

Suppose that you heard someone say the following: 'It was a great party. Everybody was really nice. Some of them even stayed after and helped us clean up.' Most people would interpret this to mean that not everybody who went to the party stayed after to help clean. But not a trained logician. He would gleefully point out that 'Some S are P' does not necessarily imply 'Some S are not P'. He would give as a counterexample something like the following: 'If it is true that some dogs are mammals (and technically it is, since 'some' is defined as 'at least one') does that mean that some dogs are not mammals?' Obviously it would not, because as everyone knows, it is actually the case that all dogs are mammals. So, even though the speaker said 'some', for all we know it could have been everybody.

Good thing the party is over, because if it was not, this is where it would take a turn for the worse, as I would have to take issue with the logician. I would ask him why he thought that the speaker would use the word 'some' if what she really meant was 'all'. If she knew that everybody stayed to clean why wouldn't she just say that?

It seems to me that the logician is not playing by the rules. He is using terms in a different way than how they are used and understood outside of logic. In standard everyday discourse people do not ever use the term 'some' to mean 'all', so for them 'some' does imply 'some are not'. As far as his counterexample, I would ask why anybody would want to make the claim 'Some dogs are mammals' and whether that is actually even true. It is true that at least one dog is a mammal, I'll grant him that, but is 'at least one' really the correct definition of 'some'?

Imagine that you were on a ship and when you saw some humpback whales another tourist nudged you and said: 'Did you know that some whales are actually mammals?' Wouldn't you feel the need to gently correct him and say: 'Uh, well actually all whales are mammals.' Don't you think that somebody else would probably say it if you didn't? I think it would actually be very odd if everybody just agreed with him. I mean, at the very least, his statement seems imprecise. Why not just say 'all' if it is actually all? It is kind of like saying that in basketball the team that scores the most points usually wins, or sometimes two plus two equals four. And speaking of the word 'sometimes', we never use that to mean 'all times' now do we?

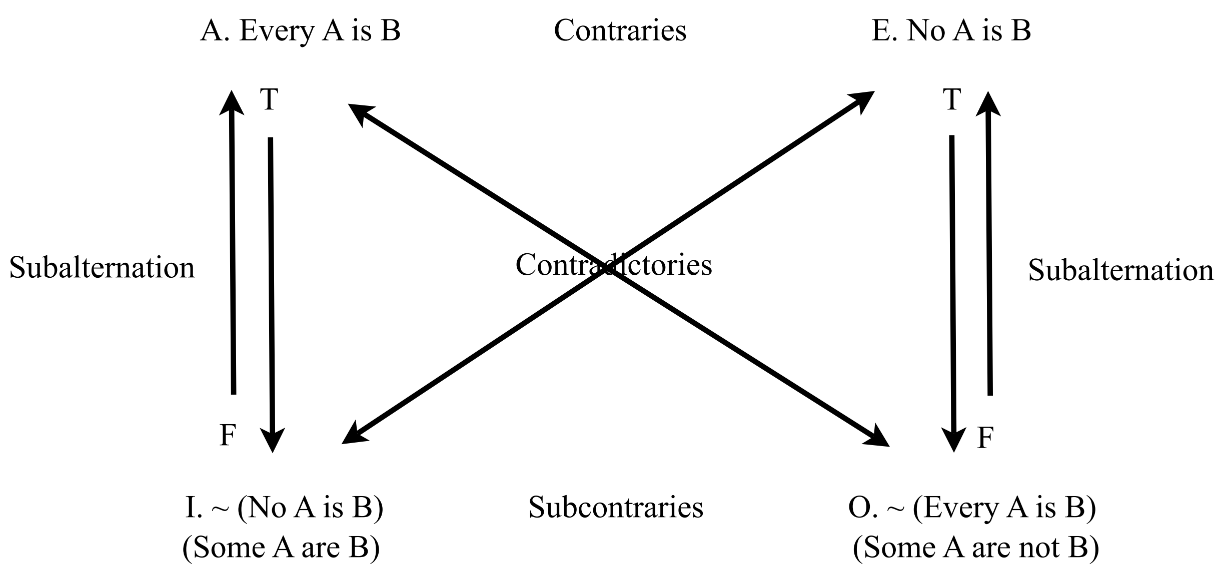
The way that the logician uses the word, 'some' could also mean none. This happens with negative claims. 'Some S are not P' is defined as 'there is at least one S that is not P', which would be fulfilled if no S are P, so 'Some snakes are not elephants' is considered true. But if you say that the next time you are visiting the zoo everybody will think you are crazier than the chimpanzees.

There is way too much ambiguity here. For goodness sakes, just say what you actually mean! If it is all or none, then say so. Now maybe there could be times when there is some uncertainty about whether the claim should be universal or not, but if so, one could just say: 'It seems as though all S would be P' or 'It is probably the case that no S are P'. Nothing could be deduced from such propositions until more information was discovered empirically, but at least they would be clearly communicated.

A better definition of 'some' is 'not all and not none' which is how most people use it anyway. Based upon this definition, in order for 'Some S are P' to be true, there has to be at least one S that is a P, and at least one S that is not. In other words, part of the subject class has the predicate, and the other part does not.

The Square of Opposition

This definition of 'some' would drastically alter the logical relations of the Square of Opposition. First I will explain those relations, then we will discuss the alterations.



The relationship between the A proposition, or 'Every A is B', and E, or 'No A is B', is that of polar opposition, meaning that they are as far apart from each other as they can be. Obviously both of them could not be true at the same time, but it is possible for both to be false at the same time. 'All hurricanes form in the Atlantic Ocean' and 'No hurricanes form in the Atlantic Ocean' are both false if in fact some hurricanes form in the Atlantic Ocean and some form elsewhere. This is known as the *contrary* relation.

Suppose that you knew the claim 'It is hot outside' was false; what else would you know? You may be tempted to say 'It is cold outside' must then be true, but that is not necessarily the case. It could be neither one. 'Cold' is the polar opposite of 'hot', but its logical opposite is simply 'not hot'. This could mean 'cold', but it does not have to: 'moderate', 'a bit chilly', or even 'slightly warm' would also count as 'not hot'. If it is one of these, then it would be false that it is hot and false that it is cold. In this case, since we already know that 'It is hot outside' is false, we can conclude that 'It is not hot outside' must be true. If, on the other hand, 'It is hot outside' were true, then 'It is not hot outside' would have to be false. A and O are *contradictories*, meaning that they always have opposite truth values. E and I are also contradictories. If 'Some dogs are brown' is true then it has to be false that 'No dogs are brown'.

It is not only possible, but necessary that if 'It is hot outside' is true, it must also be the case that 'It is not cold outside' is true. In other words, if 'It is hot' is true then the contradictory of 'It is cold' must also be true. I is the *subaltern* of A, and O is the *subaltern* of E. If A is true then I must be true, and if E is true then O must be true. The reasoning here is that if the universal statement is true then surely the more restricted claim about 'at least one' member would also have to be true. So, if 'No bachelors are married' then 'Some bachelors are not married' is also considered true, and if 'All men are mortal' then 'Some men are mortal' is also thought to be true.

A and E are *superalterns.* If the I statement is false then it may be inferred that A, its superaltern, is also false. If 'It is not cold outside' is false then we would know that it is cold, so 'It is hot outside' must be false. So, if 'Some bachelors are married' is false then 'No bachelors are married' must be true, and 'Every bachelor is married' must be false. Similarly, if 'Some horses are not mammals' is false, then 'No horses are mammals' is also false and 'Every horse is a mammal' is true.

Finally, as previously discussed, it is possible for something to be neither hot nor cold; if we were talking about a liquid it could be lukewarm, or room temperature, or slightly cool; in any of those cases 'It is hot' would be false, as would 'It is cold'; what is true is that it is neither hot nor cold, meaning that the proposition 'It is not hot' is true, and the proposition 'It is not cold' is also true. For the Square of Opposition, when the two universals (A and E) are both false the particulars (I and O) are both true. 'Some dogs are fast' and 'Some dogs are not fast' are both true at the same time. But I and O cannot both be false at the same time because then both universal statements would have to be true (by the contradictory relation) and that is not possible. This relationship is the inverse of the *contrary* relationship between the universals, and the name reflects it: I and O are *subcontraries*, meaning that both can be true, but they cannot both be false at the same time.

With a little reflection one will notice that there are really only three possible outcomes: either both particular statements are true, or A and I are true, or E and O are true. Sometimes we may not know which way it actually is; for example, if we only knew that the I statement was true then we could conclude that E was false, but the truth values for A and O would be unknown. But even if two of the values are unknown to us, it is only those three results that are possible.

The Triangle of Opposition

Now consider what would happen to the logical relationships of the Square of Opposition if the definition of 'some' was 'not all and not none' rather than 'at least one' . Here are some of the inferences that could be made:

If A is true: I is false, E is false, O is false

If E is true: O is false, I is false, A is false

If I is true: O is true, A is false, E is false

If O is true: I is true, E is false, A is false

These results are fairly straightforward, but it gets a little more complicated when we only know that a statement is false. For example, if A is false we might assume that O would be true based upon the contradictory relationship, but if O is true then I would have to be as well. Suppose that this false A statement is 'All dogs are reptiles'; this would mean that 'Some dogs are not reptiles' and 'Some dogs are reptiles' would both have to be true, but that is obviously incorrect. It is actually the E statement 'No dogs are reptiles' that is true, and all others are false. What this example shows is that if a universal is false one could not necessarily infer that its contradictory is true. It could be either the contradictory or the contrary that is true. You would not know which without more information. If the false universal statement was instead 'All dogs are fast' then E would also be false, while I and O would both be true.

If a particular is false we have a mirror image of the same problem. Suppose that the false statement is 'Some dogs are reptiles'. Could we know with certainty that its contradictory must be true? In this case E is true, but it would not be that way for other statements. If the claim was 'Some dogs are mammals' that is false according to our definition of 'some' as 'not all and not none' but it would not be correct to say that its contradictory 'No dogs are mammals' is true. When one particular statement is false the other would have to be as well, but it could not be deduced which universal statement is true.

The main takeaway from all of this is that contradictories would not always have opposite truth values.

If A is false: Either E is true or O and I are true.

If E is false: Either A is true or I and O are true.

If I is false: O is false and either A or E must be true.

If O is false: I is false and either A or E must be true.

These results could be generalized to the following rules:

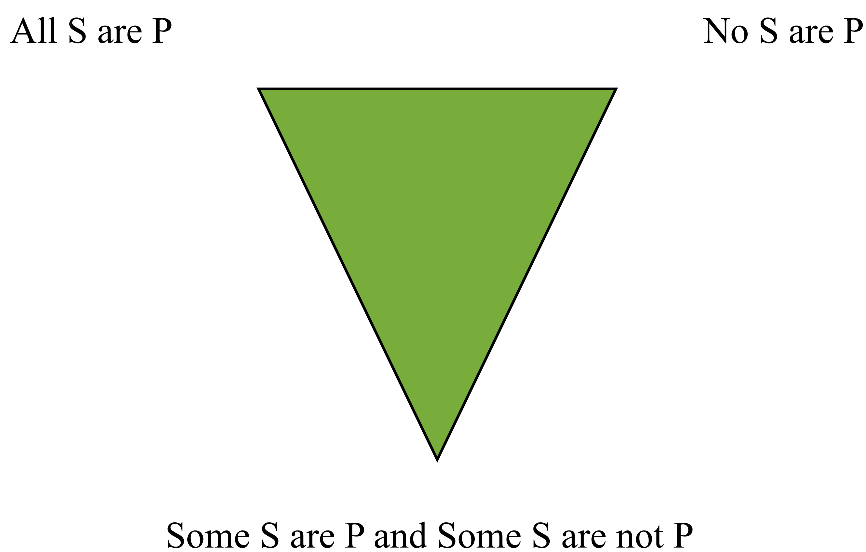
1. If either universal is true all other claims are false.

2. If one particular is true the other must also be true, and both universals must be false.

3. If one particular is false the other must be false as well, and one of the universals must be true; however there is no way to know which without more information.

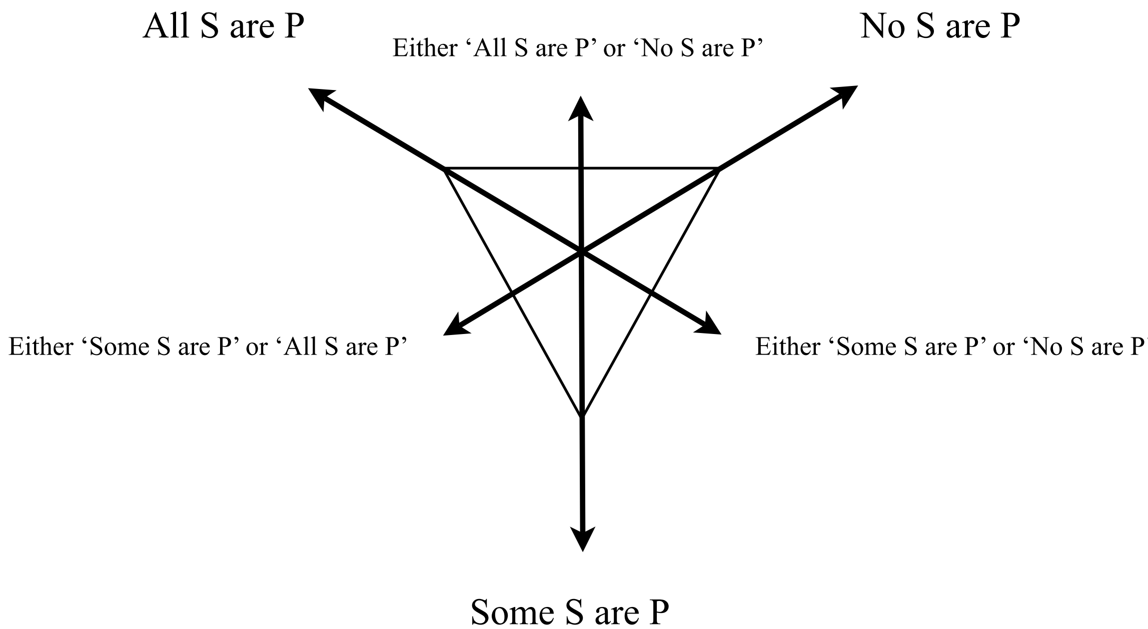
4. If one of the universals is false then either both particulars are true and the other universal is false, or the other universal is true and both particulars are false; there is no way to know which way it is without more information.

This is not too bad, but it could be simplified even more. One will notice that the truth value of the two particular statements is always the same so we could just combine I and O into one compound statement and create a Triangle of Opposition:



The number of categorical propositions is now three. This represents the full range of possibilities for the relation of S to P. One of these three statements must always be true no matter which categories S and P represent. (Although there could be times when there is not enough information about the categories to know which.) Each statement is in opposition to the others, meaning that they all have the same subject and predicate but differ in quantity or quality, so if we know which of them is true we would automatically know that the other two are false. However, if all that we are told is that one of them is false then we would know that one of the other two must be true but we could not deduce which one from the given information. If the terms represented categories that we have some familiarity with we could probably tell that way, but it would not be through logical inference.

The three statements are not *contradictories*. The contradictory relation is this:



Each contradictory statement should be understood as an 'exclusive or', meaning that part of it (one of the disjuncts) is true and the other part is false. This means that if 'All S are P' is false it must be the case that either 'Some S are P' is true, or 'No S are P' is true, but not both of them. We would not know which it was without more information. On the other hand, if 'All S are P' is true, then both sides of the disjunction are false for the contradictory, which means that the statement as a whole is false.

Since an 'exclusive or' is true when one and only one disjunct is true, two of the 'or' statements would always be true. For example, if 'No S are P' is true then 'Either "Some S are P" or "No S are P"' and 'Either "All S are P" or "No S are P"' are also true; the only 'or' statement that is false is the contradictory of 'No S are P'. However, if all that we were told is that one of the 'or' statements is true (which is equivalent to being told that its contradictory is false) we would not be able to tell which other 'or' statement is true merely through deduction. Thus, in some cases you could deduce the other five truth values from the one that you were given, but in others you would only be able to deduce the truth value of the contradictory.

Now a brief note on translations. 'All stories cannot end happily' seems to be equivalent to 'Not all stories can end happily' and 'All municipal bonds do not have the same rating' seems to be equivalent to 'Not all municipal bonds have the same rating'. The question is how one should interpret ‘not all’; technically it could mean either ‘none’ or ‘some’ because ‘not all’ seems to be equivalent to saying that ‘All S are P’ is false, so its contradictory would be true. But in practice, speakers never really use ‘not all’ to refer to none, so it should be translated as ‘some are and some are not’ rather than the contradictory of ‘All S are P’. ‘Not all’ is generally used when the author or speaker believes that others erroneously think that ‘All S are P’ is true and he or she wishes to show that it is actually only some.

S*ubalternation* is not valid for the Triangle of Opposition because with the particular statements merged together into a single proposition a universal could not imply its subaltern without also simultaneously implying its own contradictory (Aristotelian interpretation), which of course is absurd. *Superalternation* is not valid either because if the particular statement is false we would know that one of the universals is true, but we would not know which one.

This is Aristotle’s interpretation using an identity relation:

All A are A true	No A are A false	All A are non-A false	No A are non-A true
Some A are A true	Some A are not A false	Some A are non-A false	Some A are not non-A true

I guess it works if by ‘some’ you mean ‘at least some’, but isn’t it strange to say that ‘Some A are A’ and ‘Some A are not non-A’ are true? To me, this is much better:

All A are A true	No A are A false	All A are non-A false	No A are non-A true
Some but not all A are A false		Some but not all A are non-A false	

The Modern Square of Opposition also rejects the subalternation, contrary, and subcontrary relations, but for a different reason. The Modern Square is based upon what is often referred to as the ‘Boolean interpretation’, which is focused on existential import. In fact, this interpretation is obsessed with existential import, which is supposed to be the reason why none of the relations of the Traditional Square are valid except for the contradictory relation. However, there are serious problems with this interpretation.

Existential Import

According to John Venn's interpretation, universal propositions (A and E) do not necessarily imply the existence of the subject. They are equivalent to a conditional statement such as: 'If it is an S, then it is a P' for A, or: 'If there are any S, then they are not P' for E. But he interpreted particular statements (I and O) differently. Since they apply to at least one member of the subject class, he concludes that at least one member of the subject class must exist in order for a particular statement to be true. 'Some S are P' is interpreted to mean 'There exists at least one S and it is a P', and 'Some S are not P' is interpreted as 'There exists at least one S and it is not a P'. Thus, for Venn, universal statements do not have existential import, but particular statements do.

This interpretation leads to major inconsistencies and even outright absurdities. It would mean that when the subject class is empty both universal statements about it are true at the same time, even if the proposition is self-contradictory. For example, 'No unicorns are unicorns' would be considered true, as would 'All unicorns are non-unicorns'. If no unicorns are unicorns then what *would* be a unicorn? If nothing at all, then what are we referring to when we use the term? And how could all unicorns be non-unicorns? Even if there are not any real ones in the actual world, the term still has meaning. Fictional subjects often have sense without reference (in the actual world).

For Venn's interpretation any claim that one wanted to make about fictional subjects would be true as long as it is universal. Even statements like 'All things identical to Superman are female' or 'Wonder Woman is a man' or even 'Wonder Woman is an insect' would have to be considered true. But to me, it seems *a priori* that the first two are false because gender is specifically identified as part of the name or title that the subject has and the proposed predicate would contradict this. Unless their titles are complete misnomers, it is analytic that Superman would be male, and that Wonder Woman is a woman. The third statement could maybe be considered analytic as well, because if she is a woman then she is not an insect; but even if it is not considered analytic we know that it is false because of our knowledge about the fictional character.

It is also problematic to say that all particular statements about fictional subjects are always false. Is 'Some hobbits are named Frodo' really false just because hobbits do not exist in the real world? I could have sworn that I watched about ten hours worth of movie about a hobbit named Frodo. (Although it was really good, so no complaints.) Why is that not considered true relative to Middle Earth and/or the *Lord of the Rings* story? There is a hobbit named Frodo!

'Some Vulcans have two eyes' is false for Venn because the category is empty in the real world and there has to be at least one member for it to be true. But 'All things identical to Spock have two eyes' would be considered true because it is universal. Now tell me, how exactly can it be true that Spock, a Vulcan, has two eyes, and at the same time false that there is at least one

Vulcan that has two eyes? I think Spock himself would be very disappointed in us for coming up with logic like this.

Although it is often referred to as the ‘Boolean interpretation’, I do not believe that George Boole was actually even committed to this view that is so often attributed to him. Boole’s goal was to show how the four traditional categorical propositions could be rendered as algebraic equations. To do so, he used 1, or unity, to represent the ‘universe’ of all possible classes. Boole did not mean the physical universe, he defined this ‘universe’ as: ‘every conceivable class of objects whether actually existing or not.’

It is probably this latter part of the phrase - *whether actually existing or not* - which is largely responsible for the belief that for Boole universal propositions do not have existential import, and actually he probably did have that view. But there is no reason to suppose that Boole thought that particular propositions do necessarily have existential import, and this is where his view differs from Venn and the ‘modern interpretation’ that follows Venn. If Boole’s universe is comprised of every conceivable class of objects, some of which actually exist and some that do not, then why suppose that a particular statement would automatically be considered false for him if it was about a subject class that does not actually exist? It seems to me that his true position would be that none of the four categorical propositions necessarily have existential import. He just wanted to give an algebraic expression that is as broad as possible, and he recognized that the same expression should work whether the classes have actual members or not. He was really only concerned with class inclusion and exclusion, not whether those classes have actual members; in fact, by defining his ‘universe’ in the way that he does, he seems to be specifically trying to get away from having to deal with the issue of existential import.

Interpreting Boole in this way helps to explain why he argued that an A statement allows for conversion by limitation (‘All Xs are Ys’ implies ‘Some Ys are Xs’). As a Venn diagram would indicate, conversion by limitation is not valid for the so-called ‘Boolean interpretation’ because for Venn universal propositions do not have existential import whereas particular statements do. But it would be valid if none of the four propositions have existential import; it works if all the statements have existential import or if none of them do. Boole did seem to be assuming that the categories had members, but those members would not necessarily have to be actual, and that would be the case for any of the four statements.

As for my view, I believe that if the class has no actual members then all three categorical propositions have no truth value relative to the actual world. All of them would simply be undefined or inapplicable to the real world. A subject that does not exist does not have any actual characteristics or attributes, and any speculation about the characteristics that it would have if it existed is merely hypothetical. But a proposition about a fictional subject could have a hypothetical truth value, and many of them do.

Some argue that the universal negative (E) does not presuppose the existence of members of the subject category, but clearly it is meant to refer to two categories that do have actual members in which there is no overlap between them, such as ‘No dogs are cats’. If there were not any dogs, what would you be referring to? It would be a proposition about nothing, or ‘no-thing’, which does not make any sense. Moreover, it leads to self-contradictions to say that the E statement is always true when the subject class has no actual members. What about ‘No unicorns are one-horned creatures’ and ‘No things identical to Superman are male’, and similar claims?

Because a predicate can be considered a characteristic or attribute of the subject, as well as another category, I would be willing to go along with the idea that it is not required for the predicate category to have actual members for the statement to have an actual truth value. The subject obviously does not have that attribute or characteristic if the predicate category does not have any actual members, so ‘No S are P’ is true and the other two are false. For example, ‘No dogs are unicorns’ is true because no dogs have that attribute, or are members of that class. But if ‘unicorns’ is the subject none of the three would have any truth value relative to the actual world.

Although a proposition about a hypothetical or fictional subject’s attributes is just conjecture, sometimes it is obvious what its truth value would be if the subject did exist, such as ‘No unicorns are triceratops’. It is *a priori* that this would be true if there were unicorns, but of course it is not realized as an actual fact in the real world because it does not refer to an actual thing in the real world. ‘If there were unicorns then all unicorns would be unicorns’ is of course hypothetically entailed. ‘Based upon the supposition that there were unicorns all unicorns would be unicorns’ is true, but the supposition is not a fact, it is actually a counterfactual, so the claim is hypothetically true, but not actually true. Obviously only a hypothetical conclusion could be drawn from a hypothetical premise, even if the inference is valid. To avoid confusion one should use a hypothetical claim for a hypothetical subject, such as ‘If there were . . .’. This allows us to add the condition that it would only really be so if the class had members, which shows that one is just entertaining a counterfactual.

Another way of approaching the issue would be to confine the statement to a fictional context. However, one must clearly stipulate that it is meant to refer to this context only, not the actual world. An example of this would be: ‘In Irish folklore, a leprechaun is a small mischievous sprite.’ This is true. It is not committed to saying that leprechauns actually exist.

If it has been clearly established that a claim refers to a fictional setting, such as the *Star Wars* Universe, then it could have a truth value relative to that fictional world, but of course it does not apply to the actual world. An interesting implication of this is that ‘All things identical to Yoda are wise’ has an actual truth value within the *Star Wars* universe while ‘All things identical to Socrates are wise’ would have only a hypothetical truth value there, because of course it is possible for Socrates to exist in the *Star Wars* universe, but so far as we know, he does not actually exist there. The principle that can be taken from this is that if a subject does not exist in a world then propositions about it do not have an actual truth value there. (Fictional settings would be included in possible worlds, and most possible worlds would probably see themselves as ‘the actual world’.)

There is one exception to all of this. A categorical statement about a fictional subject does have an actual truth value if the predicate is existence. ‘All unicorns are merely fictional creations’ or ‘Dodo birds no longer exist’ are both true in actuality, even though the subject classes do not have any actual members (at least not presently in the case of Dodo birds) because that is the very fact that the proposition is asserting. Both of these statements are equivalent to saying that the subject is not a member of the class of existing things, or that existence is not one of the subject’s predicates. In the case of Dodo birds, the class members once had existence as a predicate, but not any longer. ‘Dogs exist’ correctly indicates that the ‘dogs’ class has the predicate of existence. ‘Leprechauns exist only hypothetically’ is saying that the subject is not a member of the category of ‘things with actual existence’ and/or that it is a member of the category of ‘things that are hypothetical’, which of course is true. This is different than ‘All leprechauns are small’ or ‘No leprechauns are tall’. The subject cannot possess (or lack) any other actual attributes if it does not have actual existence. The subject is located in the class of ‘fictional things’ so it has predicates there, but it is not found in the ‘actually existing things’ class at all, so such claims are undefined for the actual world.

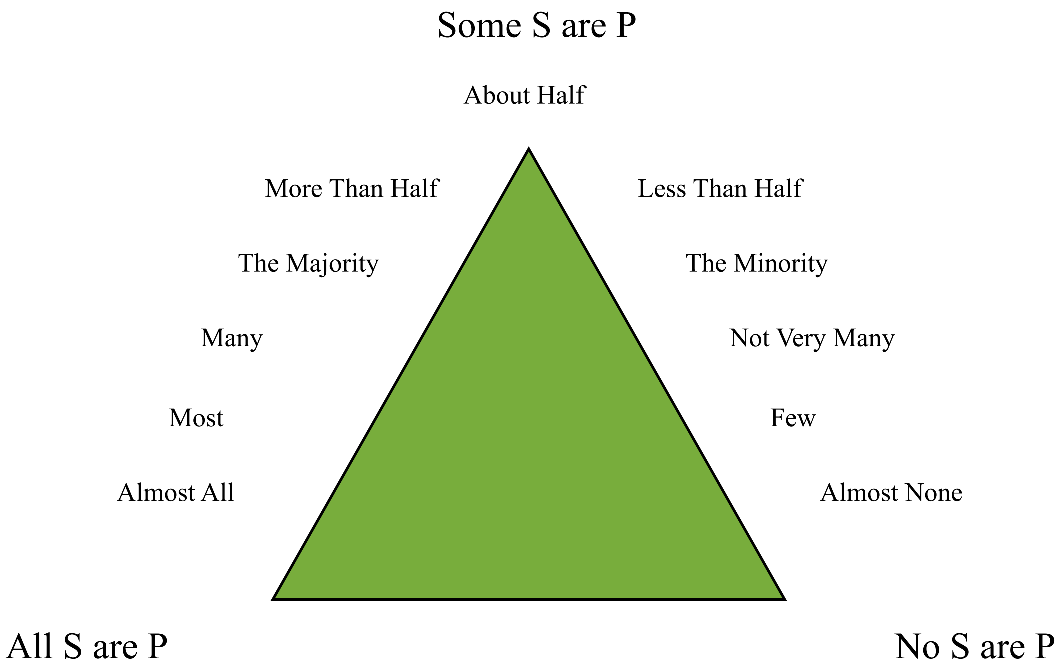
Enough about existential import. In addition to what I have said here, I have already discussed the issue extensively elsewhere, so I am ready to move on. Universals require at least one actual member of the subject class to have a truth value for the actual world, but if one interprets singular propositions as I will suggest below, then universal statements would nearly always be about subject classes that have more than one member. (The A statement also requires at least one actual member for the predicate class; typically it would have a lot more.) The particular statement would be false if there is only one actual member of the subject category; if it is true there would have to be at least two members, and at least one member of the predicate category, because there would have to be at least one member of S that is a P (which requires at least one S and one P), and at least one other member of S that is not a P. This is the only true difference in the existential import of the propositions.

The ‘Some’ Spectrum

Consider quantifiers such as ‘few’, ‘many’, ‘most’, ‘almost all’, ‘almost none’, ‘the majority’, ‘the minority’, ‘more than half’, etc. None of these could mean ‘all’, for if it is ‘almost all’ (or ‘almost none’) then clearly it is not ‘all’.

In the past it has been problematic to translate statements that use these quantifiers into categorical form because they are not really equivalent to one of the four standard categorical propositions when using the prior definition of ‘some’. One would have to translate statements like ‘Many S are P’ or ‘A few S are P’ not as an I or an O statement, but as implying both at the same time. Translating them this way is correct, but it does not result in a standard form categorical syllogism, so one could then only check for validity using extended techniques, and actually I do not believe that most of the attempts to do it with extended techniques are even done correctly.

All of these quantifiers are equivalent to my definition of ‘some’, they are just more specific versions of it. Each of them could be translated into categorical form as ‘not all and not none’ or ‘some are, and some are not’. There is a whole spectrum of terms that could stand for ‘some’:



‘All’ is equivalent to 1, ‘None’ is equivalent to 0, and ‘Some’ is equivalent to a fraction or decimal point that is greater than zero but less than one. (0 < Some < 1.) This is also the percentage of S that is P. ‘Some’ is a generic term with an indefinite value, so it could stand for any of these more specific quantifiers, and others as well.

The Relationship of ‘Some S are P’ to ‘Some S are not P’

If ‘Some S are P’ always implies ‘Some S are not P’, and vice versa, a question arises: What exactly is the logical relationship between them? They are not subcontraries because they can both be false at the same time, and in fact they would be whenever one of the universals is true. Is it simply that they both imply each other? Are they both necessary and sufficient for the other? Since they always have the same truth value, one may even be tempted to say that they are logically equivalent. But of course that cannot be correct because they are saying opposite things.

Suppose that we have two ‘exclusive or’ propositions, (p v q) and (~p v ~q). They have equivalent truth tables because they are each true when one disjunct is true, and false when both disjuncts are true or both are false. The truth values are reversed except under the main operator.

PQ	P v Q	~P v ~Q	(P v Q) ^ (~P v ~Q)
1. TT	T(**F**)T	FT(**F**)FT	T(F)T **F** FT(F)FT
2. TF	T(**T**)F	FT(**T**)TF	T(T)F **T** FT(T)TF
3. FT	F(**T**)T	TF(**T**)FT	F(T)T **T** TF(T)FT
4. FF	F(**F**)F	TF(**F**)TF	F(F)F **F** TF(F)TF

What this shows is that statements can have the same truth value under the main operator without necessarily being logically equivalent. To be logically equivalent they must also mean roughly the same thing, just expressed in a different way. That is clearly not the case here. In fact they are making opposite claims, almost like reciprocal numbers. Not opposite in the sense of being contradictory (if that were true they would have opposite truth values under the main operator for each line of the table), but opposite as far as what they are claiming. I think of these as inverse statements.

Suppose that P v Q is true. Well, since one disjunct has to be true, but both cannot be true at the same time, that would mean that ~P v ~Q would also have to be true. If P is true, then Q must not be, and if Q is true then P must not be. So if one either/or statement is true, we know that the other one must be as well.

I think it is the same way with ‘Some S are P’ and ‘Some S are not P’. It is not so much that one implies or entails the other, it is that they are different aspects of the same claim, like two sides of a coin. Or how ‘convex’ and ‘concave’ could both be used to describe the same outline or surface, it would just depend on which side of it you are referring to.

Since inverse statements always have the same truth value one could conjoin them and the conjunctive statement will also have the same truth value. (See truth tables above.) When the compound statement is true both parts of it are true, and when it is not both parts are false. It is really all the same claim, it just depends on how you look at it.

You could say that one ‘some’ statement entails the other, but really it would just be a matter of choosing whether to state it as the full conjunctive statement or leave one part implied. We often do the latter because it is more convenient and it is usually not really necessary to state them both. If a person gets a 70% on a test we would usually only say that they got a 70%, not that he or she got 70% correct and 30% wrong, even though the latter is the full meaning. Since that is obvious, it is not usually necessary to make it explicit. Similarly, every ‘some’ statement could be stated as ‘Some S are P and Some S are not P’ but sometimes for greater convenience we would only state part of it and allow the other part to be implied. Like reciprocal numbers in math, together they add up to 1, or both I and O together account for all members of the subject class, just as the other two categorical statements do.

Categorical Syllogisms

I was interested to see how defining ‘some’ as ‘not all and not none’ would affect categorical syllogisms. One way that I tested it was to modify Venn diagrams by simply placing two Xs rather than one in the appropriate locations for particular premises. That works to an extent, but it is not ideal. For one thing, Venn diagrams are based upon Venn’s interpretation of existential import. They also do not show one of the forms which I consider to be valid as valid. So I have developed an alternative way of diagramming.

First things first, though: A, E, I, O are letter names that correspond to the first four vowels of the Roman alphabet and came to be associated with the categorical propositions during the Middle Ages. While we are at it, we may as well update the letters that are used to represent the statements:

A: All S are P
B: Some S are P
C: No S are P

Since for this interpretation there are three kinds of categorical proposition, and three categorical propositions are in a syllogism, there are 27 different moods. There are still four figures (which is determined by the location of the two occurrences of the middle term in the premises), which means that there are 108 forms in total.

I have checked them all and found the following to be valid:

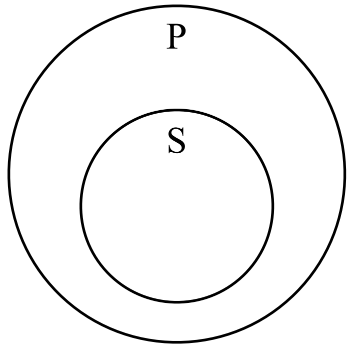
Fig 1	Fig 2	Fig 3	Fig 4
AAA		BAB	AAB
	ACC		ACC
CAC	CAC		

This list may seem sparse, but remember that this is out of 108 forms rather than 256. This interpretation is more restrictive than Aristotle’s, in which 24 forms are valid, which is roughly 9.4%. But it is actually a bit less restrictive than the ‘Boolean interpretation’ in which 15 forms are valid out of 256, which is roughly 5.9%. For me, 7 forms are valid out of 108, which is approximately 6.5%.

Here is how each type of categorical proposition is to be diagrammed and the reasoning for it:

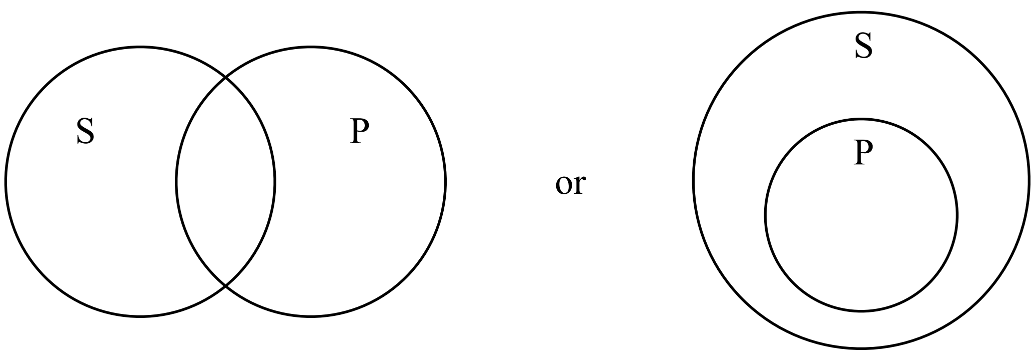
All S are P

Assuming that S and P are not synonymous terms such that all S are P and all P are S (in which case they would have exactly the same members and would just be the same class called by a different name), if all S are P then S has to be a subclass of P. There cannot be any members of S that are outside of P, so the S circle must be entirely contained within the P circle.



Some S are P

There are two possibilities. We must consider both of them when testing for validity. For the first, they are merely overlapping classes; for the second, P is a subclass of S, as in: 'Some animals are dogs'.

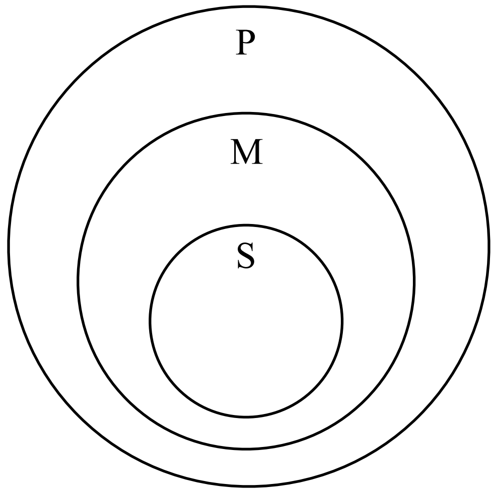


No S are P

In this case the two classes are entirely separate, so we simply have two circles that do not overlap.

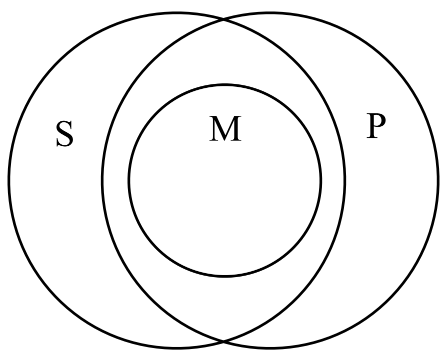
The first valid form on the list is AAA-1, commonly known as 'Barbara'.

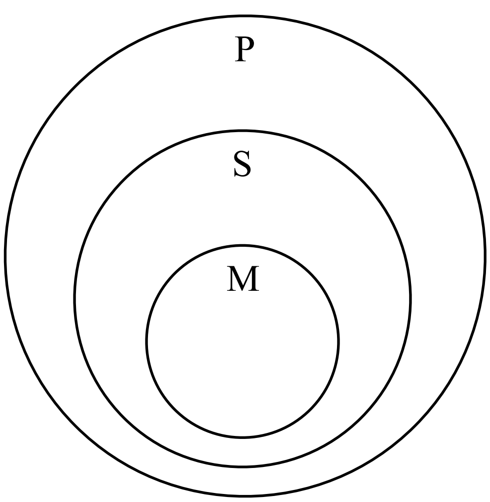
All M are P
<u>All S are M</u>
All S are P



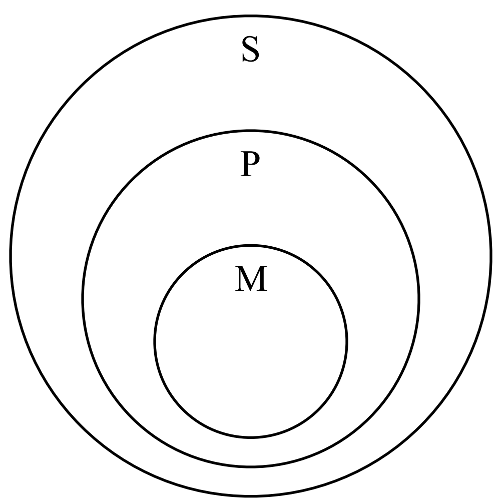
If all S are M, and all M are P, then it has to be the case that all S are P. But the other figures for AAA are not valid. For example, Figure 3 has at least three possible diagrams:

All M are P
<u>All M are S</u>
All S are P

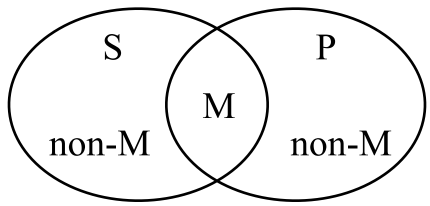




or



All three diagrams are consistent with the premises but in two of them ‘Some S are P’ would be the correct conclusion. Thus, the AAA-3 form is invalid because the conclusion ‘All S are P’ does not necessarily follow from the premises. The AAB-3 and AAC-3 forms are also invalid. These forms have identical diagrams because the premises are the same. The diagrams show that it would be valid to conclude that ‘Either All S are P or Some S are P’ is true for these premises. ‘No S are P’ is not possible; if all M are P, and all M are S, then at least some S are P where they are both M.



As one may have noticed, I do not use an ‘X’ to diagram ‘Some S are P’. However, I do find it useful for diagramming singular propositions.

The classic example of a valid argument is:

All men are mortal
<u>Socrates is a man</u>
Therefore Socrates is mortal

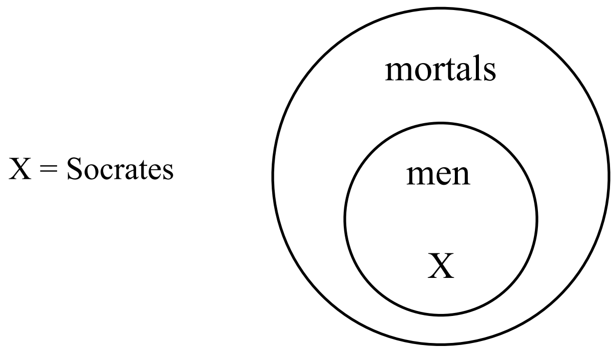
This is similar to the AAA-1, or ‘Barbara’ form. It is usually translated into categorical form as:

All men are mortal
<u>All things identical to Socrates are men</u>
All things identical to Socrates are mortal

Singular propositions like the second premise and the conclusion are treated as universals, and correctly so in my view. The category ‘things identical to Socrates’ has only one member, so it has to be universal because either all members of the subject class (of one) are members of P, and/or has the predicate, or not. One could not interpret the second premise as a particular statement because obviously it is not the case that some things identical to Socrates are not men.

Treating the second premise as an A statement means that for Venn diagrams one would shade out all parts of the ‘things identical to Socrates’ circle except for where it overlaps the ‘men’ circle, and this seems to be the motivation for translating it with that rather awkward phrase ‘things identical to Socrates’. One simply creates a class with one member so that the argument can be evaluated as a syllogism. But I think there is a better way. You could just consider the second premise to be asserting that Socrates is a member of the ‘men’ class. If we look at it that way then the first argument would not need any further translation.

It could be diagrammed like this:



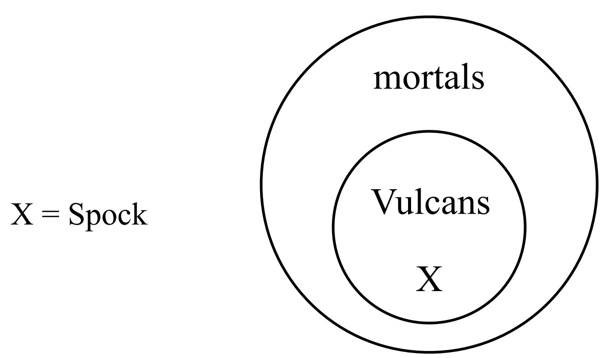
If all men are mortal, and Socrates is a member of the ‘men’ class, then he must also be a member of the ‘mortals’ class.

Of course one could consider Socrates to be his own class, in which case he would be a subclass of the other two classes and the diagram would look just like it does for AAA-1, but it is a little bit weird to have a class with only a single member. It is not necessarily incorrect, but a class or category is usually a group of things. And ‘All things identical to Socrates’ and similar phrases really are kind of awkward. Doing it this way is more intuitive and allows us to get rid of that awkward phrasing.

I realize that I am using ‘X’ in a much different way than how it is used for Venn diagrams. To be clear, it does not mean ‘some’ for my interpretation, it stands for an individual thing that has been named in the premises. I do not consider the ‘X’ to necessarily have existential import, as you could be diagramming a singular proposition about a thing that is not actual.

As a matter of fact, we could make the same argument about Spock:

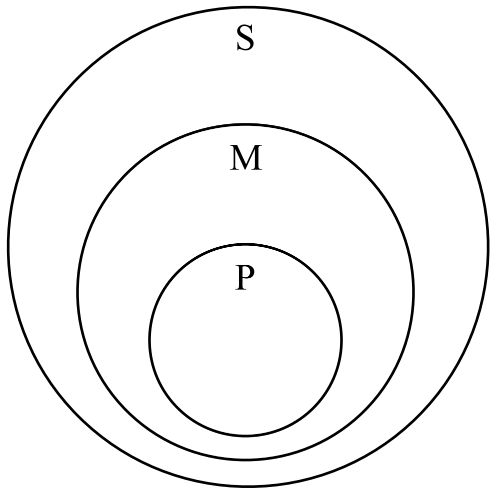
All Vulcans are mortal
<u>Spock is a Vulcan</u>
Spock is mortal

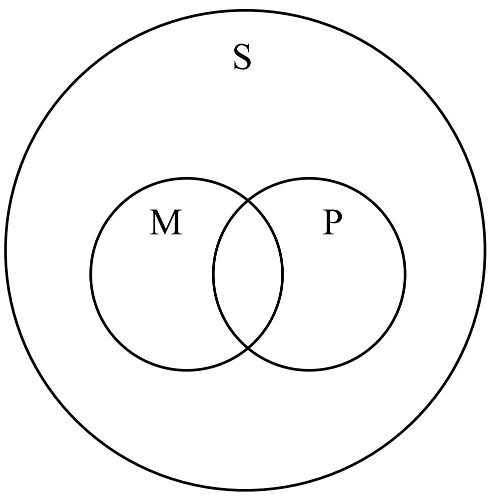


This is valid, although it does not apply to anything in the actual world. If the argument is about something that does not actually exist then it would only be hypothetical. The ‘X’ is just meant to show where that particular thing is located in terms of the classes.

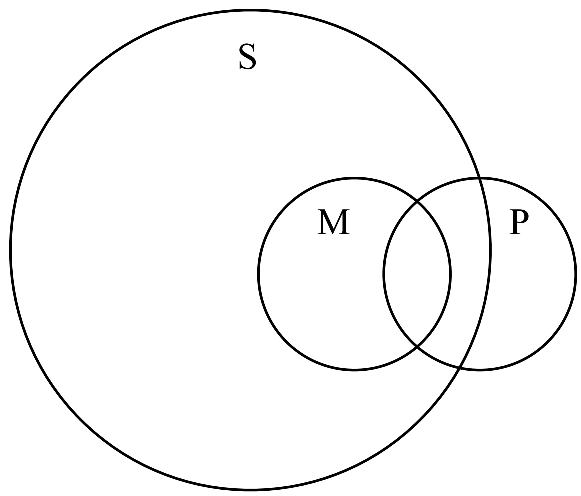
The next valid form I wish to consider is BAB-3. There are at least three possible diagrams for the premises:

Some M are P
<u>All M are S</u>
Some S are P

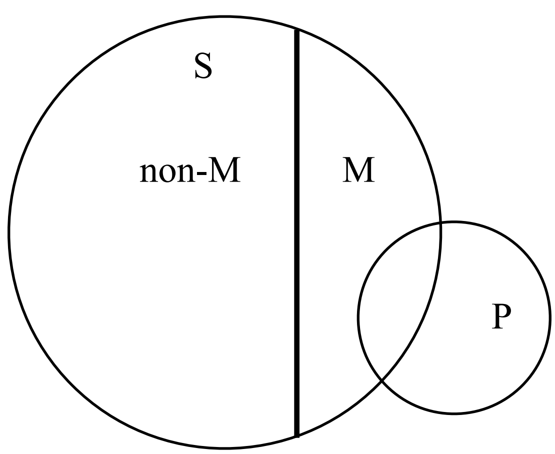




or



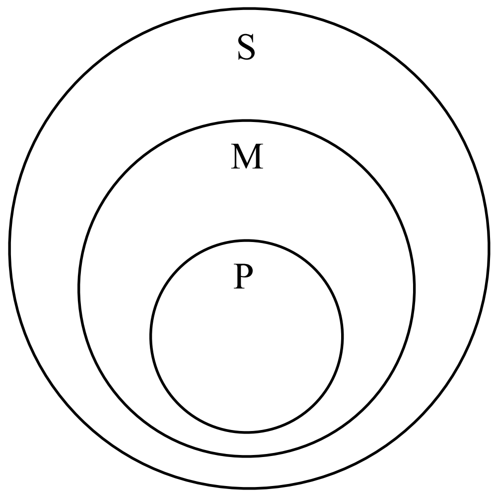
Even though there are multiple diagrams that are possible with these premises, in all of them it is true that ‘Some S are P’. Because some M are P, and all of M is entirely inside of S, it must be the case that some S are P. One could also diagram it this way:



Since some but not all M are P, it would have to be the case that some S are P (some of the ones that are M would also be P) and some are not (which would include the S that are not M, and those that are M but not P). In any possible scenario it has to be the case that some but not all S are P, which is why the argument is valid.

The next valid form I would like to discuss is AAB-4:

All P are M
All M are S
Some S are P



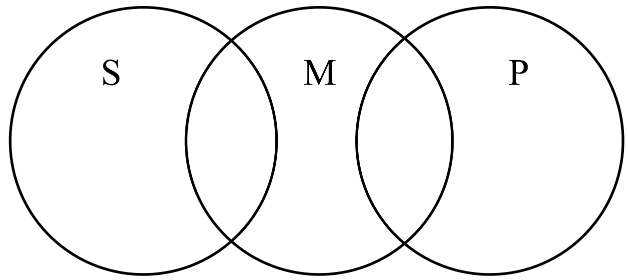
One will notice that this is the same diagram as one of the diagrams for BAB-3. It is the only one possible for these premises. The 'Boolean interpretation' is that universal premises do not imply a particular conclusion because universal propositions do not have existential import while particular propositions do; in fact, this is sometimes called the 'existential fallacy'. Sometimes this, and other forms are considered conditionally valid if you know that the class has actual members (in this case P), but Venn diagrams do not show that unless they have been modified to make an 'existential assumption'.

It should be considered a valid form. Based upon these premises it could not be the case that all S are P or that no S are P, and it has to be one of the three. If P is a subclass of M, and M is a subclass of S, then P has to be a subclass of S, which is one form of 'Some S are P'. An example would be:

All dogs are mammals
All mammals are animals
Some animals are dogs

These two forms (BAB-3 and AAB-4) are the only ones that include a particular proposition which are valid. None of the forms having two particular premises are valid. For example, BBA-1 can be shown to be invalid with the following diagram:

Some M are P
<u>Some S are M</u>
All S are P

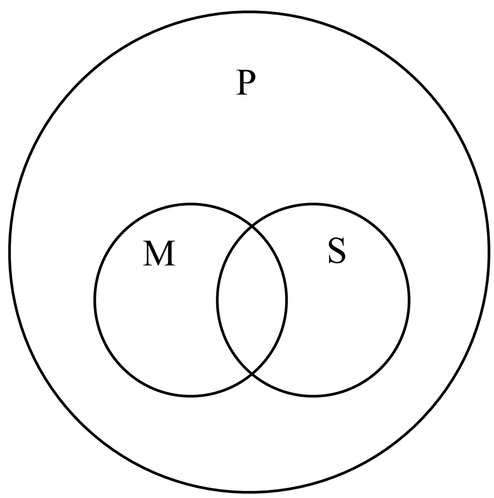


There are many other possible diagrams that would be consistent with these premises, but this one is sufficient to show that the form is invalid. In fact, this same diagram could be used to show that all four figures of BBA and all four of BBB are invalid. But it would be wrong to assume that the conclusion has to be 'No S are P'; other diagrams show that is not the case. For example, the diagram used for AAB-4 (see previous page) is also consistent with these premises. That diagram shows that 'No S are P' does not have to be true, which invalidates BBC-1.

BBC-2 and BBC-4 can be eliminated with the same diagram:

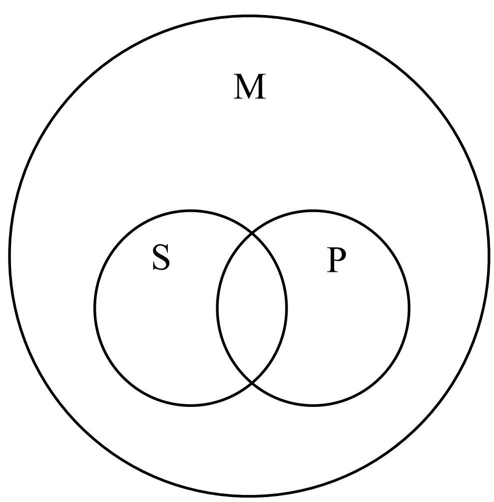
BBC-2
Some P are M
<u>Some S are M</u>
No S are P

BBC-4
Some P are M
<u>Some M are S</u>
No S are P



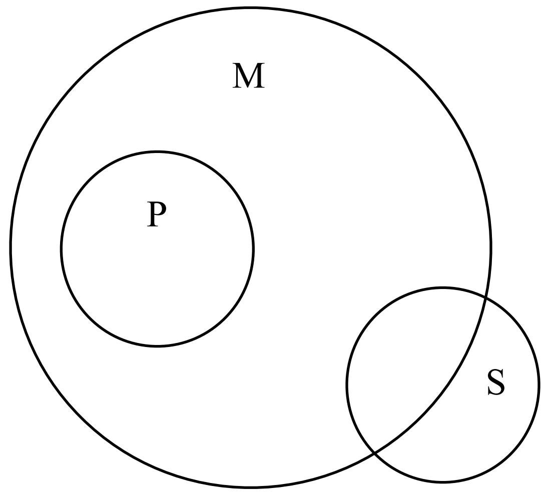
BBC-3 can be eliminated with this one:

Some M are P
<u>Some M are S</u>
No S are P

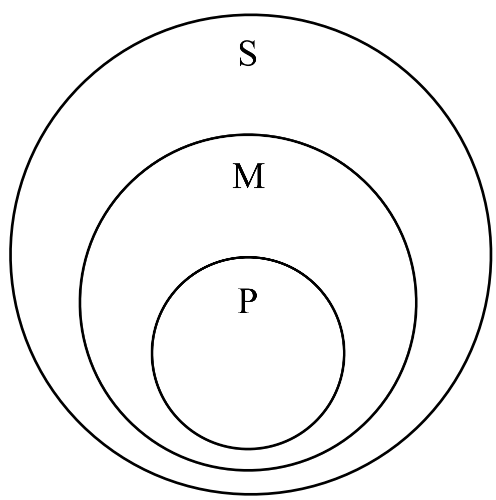


Except for BAB-3, syllogisms that have one universal premise and one particular premise are not valid either. For example, ABB-2:

All P are M
<u>Some S are M</u>
Some S are P



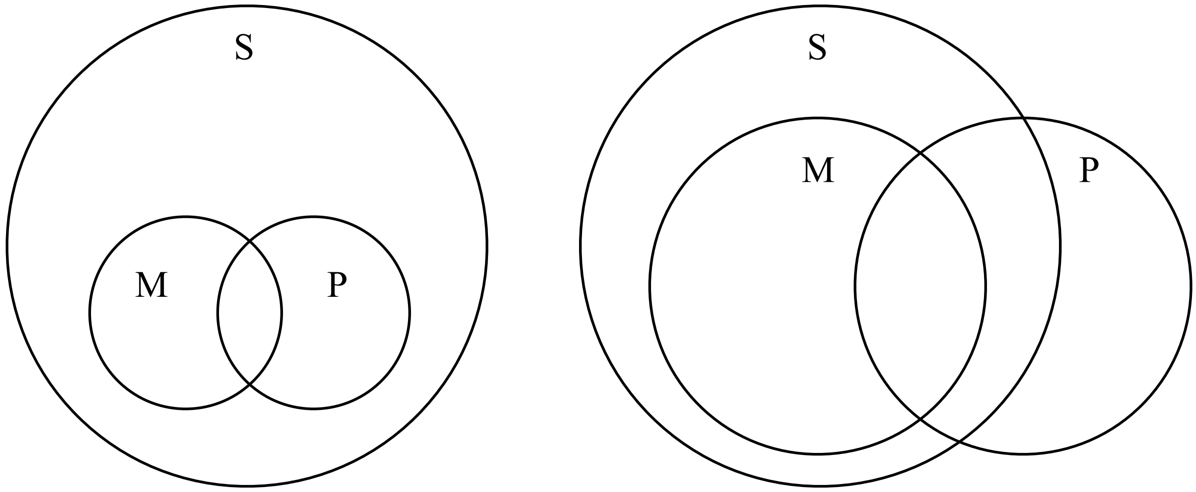
The same diagram shows that ABA-2 is invalid as well. But do not suppose that the conclusion has to be ‘No S are P’ because this diagram would also be consistent with the premises:



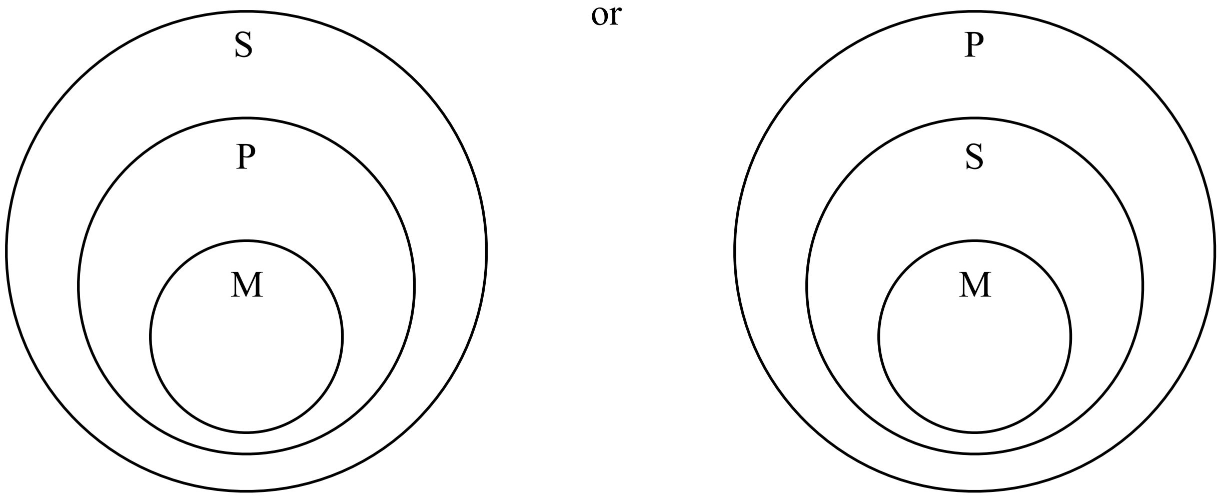
This shows that ABC-2 is also invalid.

BAB-4 has at least four diagrams that are possible:

Some P are M
<u>All M are S</u>
Some S are P



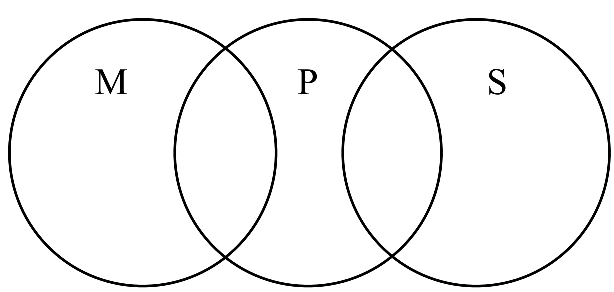
or



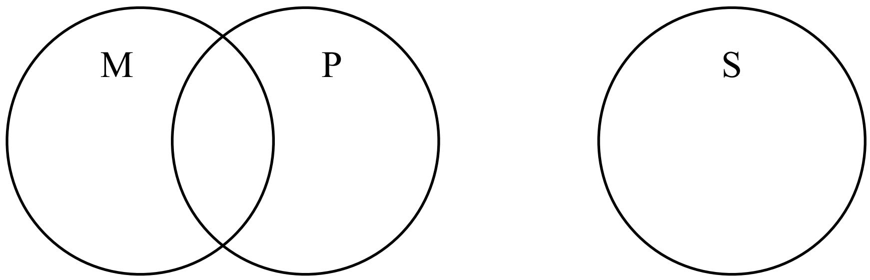
In the diagram on the bottom right, all S are P, which proves that the form is invalid. But in the other diagrams it is the case that ‘Some S are P’, which shows that BAA-4 is invalid. None of the diagrams show ‘No S are P’, so BAC-4 is also invalid. You could say that the conclusion ‘Either All S are P or Some S are P’ does follow from the premises, but of course that is not a standard form.

Forms that have a negative premise and a particular premise are not valid either. For example BCA-1:

Some M are P
<u>No S are M</u>
All S are P



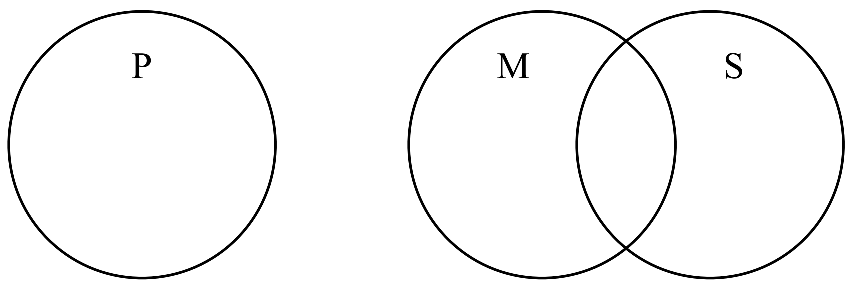
This same diagram also shows BCC-1, BCC-2, BCC-3, and BCC-4 are all invalid. But the diagram could also be drawn several other ways. For example, it could also be like this:



This diagram shows that BCB-1 is invalid. It could also be used to show that BCA-2, BCA-3, BCA-4, BCB-2, BCB-3, and BCB-4 are all invalid, although several other diagrams are possible as well.

CBB-1 is invalid:

No M are P
<u>Some but not all S are M</u>
Some but not all S are P



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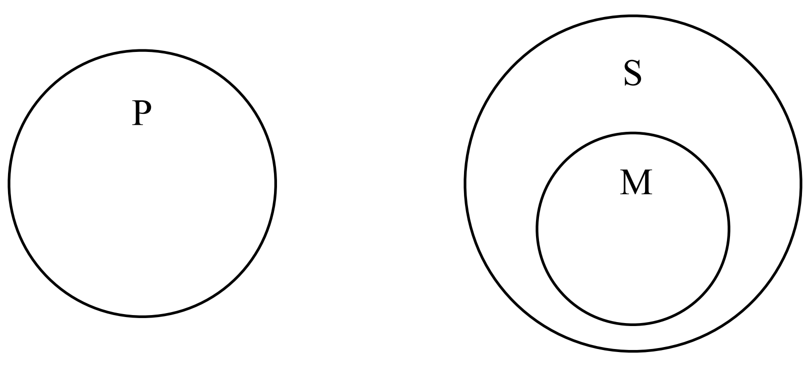
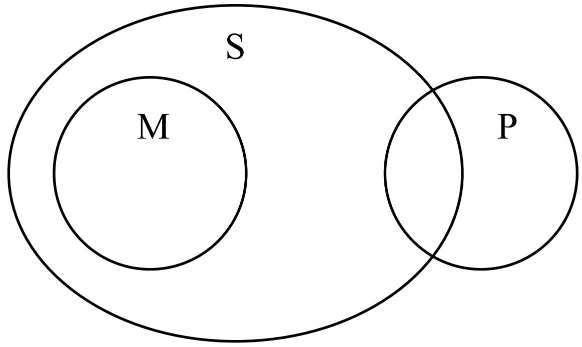


Other diagrams are possible. S could overlap P (although it could not be 'All S are P'). But these show the argument is invalid. The first diagram could actually be used to show that all four figures of CBB are invalid. It could also be used to show that all four figures of CBA are invalid.

To show that CBC-1 is invalid we could use the following diagram:

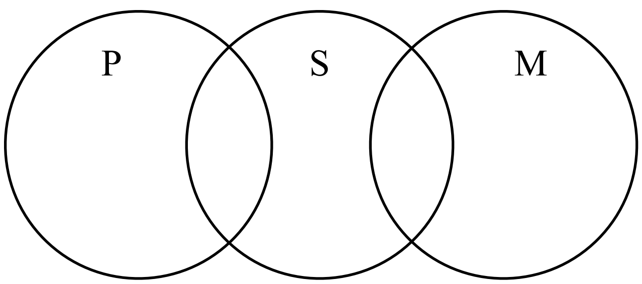
No M are P
<u>Some S are M</u>
No S are P



One could have also had the P class inside of the S class but not overlapping M.

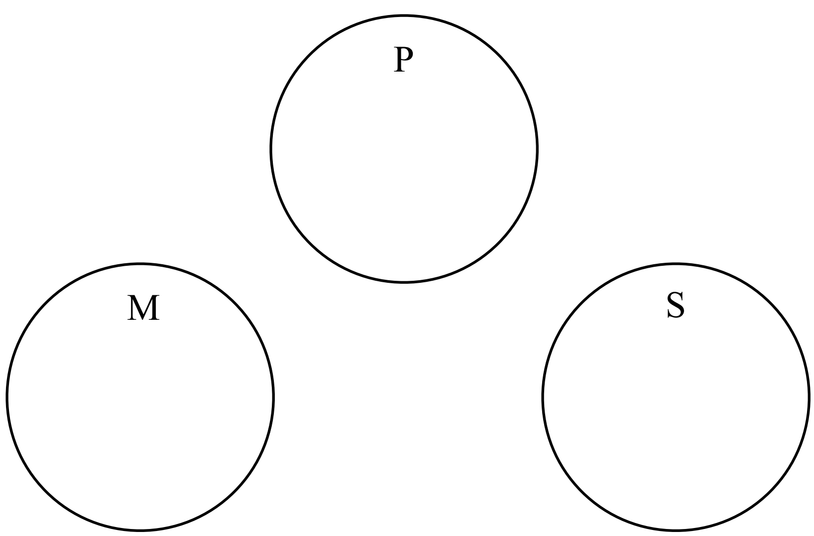
The same diagram can be used to show the invalidity of CBC-2 (the one just above works for this form as well), CBC-3, and CBC-4:

CBC-2	CBC-3	CBC-4
No P are M	No M are P	No P are M
<u>Some S are M</u>	<u>Some M are S</u>	<u>Some M are S</u>
No S are P	No S are P	No S are P



Arguments with two negative premises are invalid. For example CCA-1:

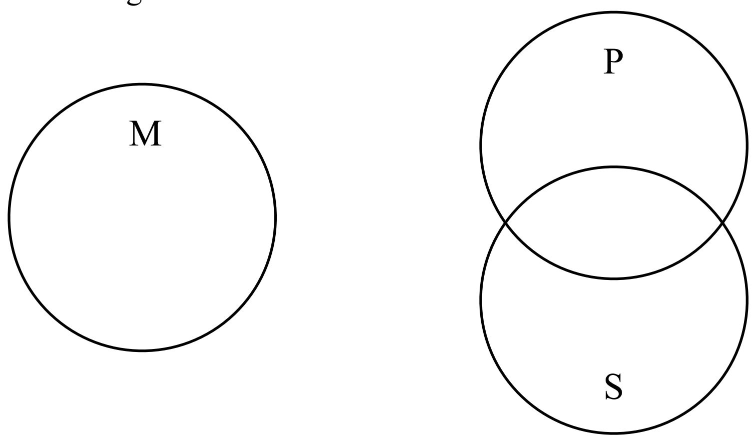
No M are P
<u>No S are M</u>
All S are P



This same diagram can be used to eliminate all four figures of CCA and all four figures of CCB.

For CCC-1 we could have this diagram:

No M are P
No S are M
No S are P

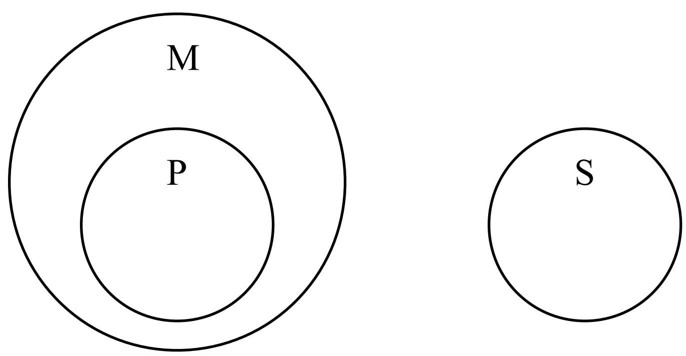


There are other possible diagrams (such as having S or P be a subclass of the other), but this arrangement eliminates all four figures of CCC.

Now we will go back to the valid forms that have not yet been discussed. ACC has two valid forms, figure 2 and figure 4. Both have the same diagram:

ACC-2
All P are M
No S are M
No S are P

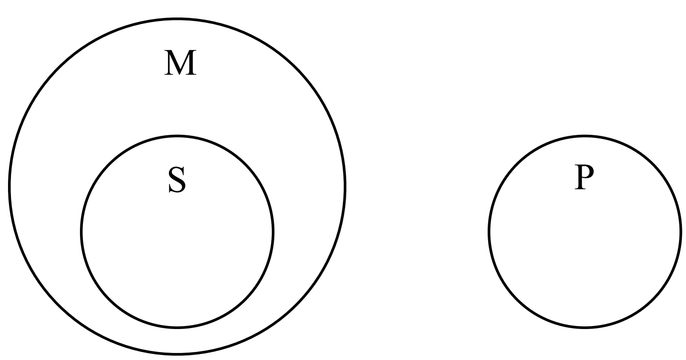
ACC-4
All P are M
No M are S
No S are P



If P is a subclass of M, and no S are M, then there is no possible way that any S could be P.

CAC-1 and CAC-2 are similar. These two forms also have the same diagram:

CAC-1
No M are P
All S are M
No S are P



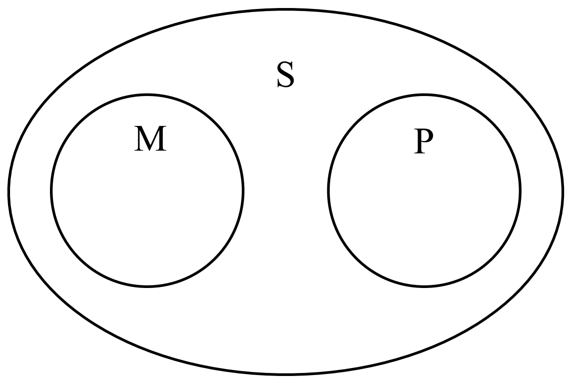
CAC-2
No P are M
<u>All S are M</u>
No S are P

Of course, since ‘No S are P’ has to follow from these premises, it eliminates CAA-1 and CAA-2, as well as CAB-1 and CAB-2 as valid forms.

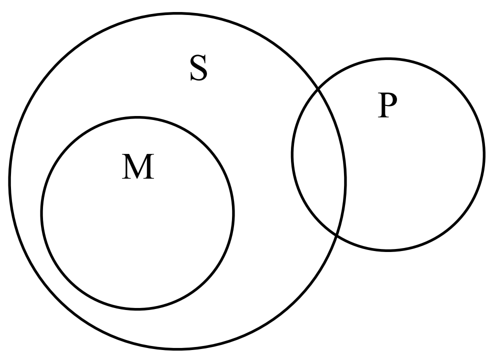
To show that CAC-3 and CAC-4 are invalid, we could have either of these diagrams:

CAC-3
No M are P
<u>All M are S</u>
No S are P

CAC-4
No P are M
<u>All M are S</u>
No S are P



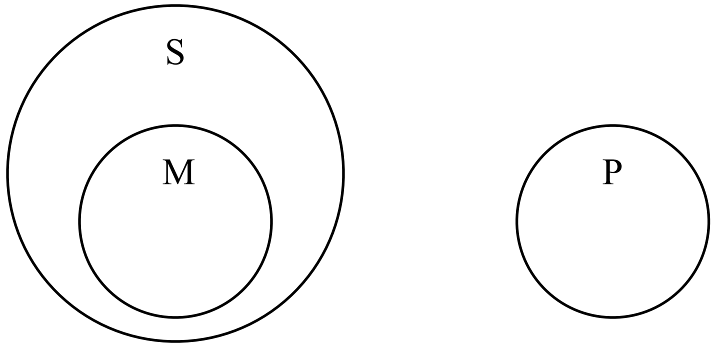
or



These two diagrams would also show that CAA-3 and CAA-4 are invalid because they obviously show that the conclusion ‘All S are P’ does not necessarily follow from the premises any more than ‘No S are P’ does. In fact, ‘All S are P’ is not even possible. But one could also diagram the premises in such a way as to show that ‘Some S are P’ does not necessarily follow either:

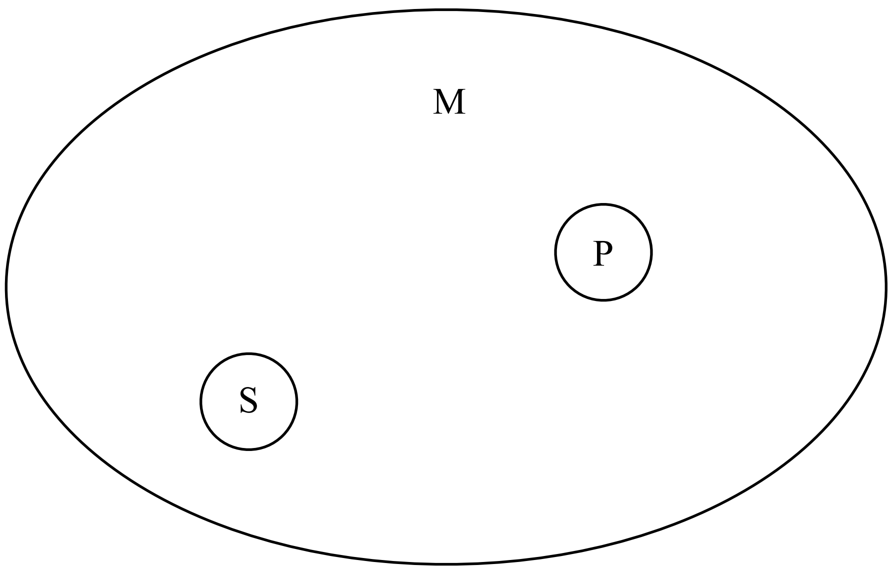
CAB-3
No M are P
<u>All M are S</u>
Some S are P

CAB-4
No P are M
<u>All M are S</u>
Some S are P



Another way of testing for validity is the rules method. Depending upon the interpretation, there are 4 or 5 rules that a syllogism must conform to if it is valid. If any one of these rules is violated a specific formal fallacy is committed and the syllogism is invalid. Probably the most well-known of these formal fallacies is the Fallacy of the Undistributed Middle. An example of this would be the following:

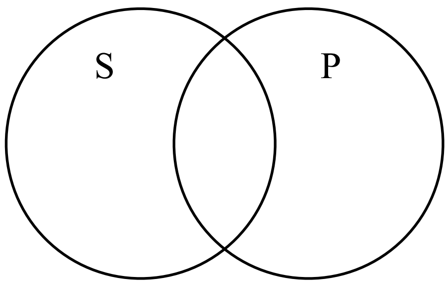
All P are M
<u>All S are M</u>
All S are P



The problem with the argument is that although we have been told that both S and P are subclasses of M we have no idea simply from this what their relation is to each other. Suppose M is a very large class, like ‘mammals’ and S stands for ‘dogs’ and P stands for ‘cats’. Well, it is true that all cats are mammals, and true that all dogs are mammals, but that does not mean that all dogs are cats. However there could be other cases where they do overlap each other, or one could be a subclass of the other (say if one class is ‘bulldogs’ and the other is ‘dogs’) so we cannot say anything with certainty about S and P merely from these premises. This is not how it is explained in terms of the fallacy, but this is the general idea of what is wrong with it.

Some of the rules hold for my interpretation, but those involving distribution are problematic. A term is considered to be distributed if the proposition makes an assertion about every member of that class. The Fallacy of the Undistributed Middle occurs when the middle term is not distributed in either premise, which is the formal reason for why it is invalid.

For both the Aristotelian and 'Boolean' interpretations neither term is distributed for the I proposition, and for O the predicate term is distributed but the subject term is not. Recall that I have two ways of diagramming 'Some S are P':



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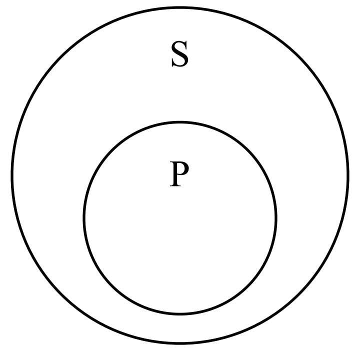


When they are merely overlapping classes obviously neither term is distributed. But when P is a subclass of S then there is a claim being made about every member of P. So for the particular proposition 'Some S are P', which terms, if any, are distributed? S is never distributed, but it seems that in one possible instance P is distributed, and in another possible instance it is not. I am not sure how you would be able to know whether P was distributed without first knowing whether P is a subclass of S or whether they are overlapping. This is a significant complication for using the first two of the rules which are about distribution.

But honestly I do not think this is really that much of a problem. It would be rather difficult to memorize 24 valid forms, which is why the rules method is useful for Aristotle's interpretation, but memorizing only 7 valid forms is actually pretty easy, especially because four of those are repeated in two figures. I have them memorized, not because I necessarily set out to do it, but just from referring to them repeatedly. I think it would be easier for most people to just come up with a mnemonic to remember the 7 valid forms rather than memorizing five rules plus which terms are distributed for each proposition. The whole concept of distribution can be a bit tricky for some people anyway, especially when they are first learning it.

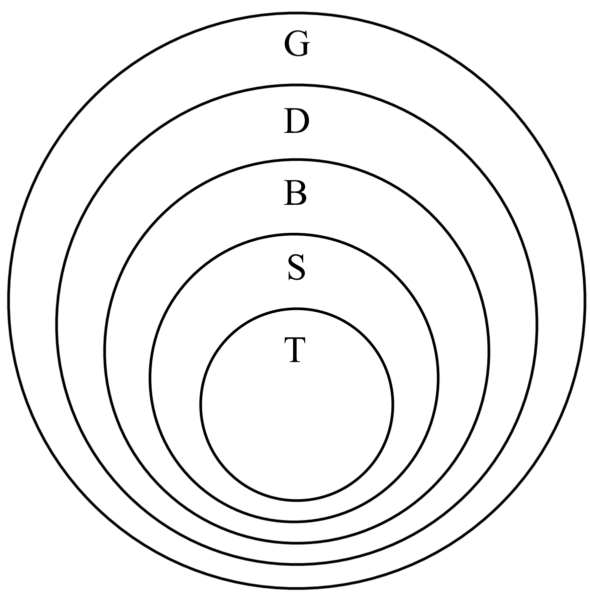
Of course it could be helpful to remember a few rules of thumb that are not related to distribution, such as arguments with two negative premises or two particular premises are always invalid, or even if there is one negative premise and one particular premise. Maybe we could simplify that even more and just say that there must be at least one A proposition in the premises to be valid, etc. But if one forgets whether a form is valid it is pretty easy to diagram it and find out, so the rules are not really necessary.

Sorites

A sorites argument is a chain of categorical syllogisms. The name is derived from the Greek word *soros,* meaning 'heap'. The method of diagramming used above is easy to extend to sorites arguments. Venn diagrams can also be used to check for validity, but it is necessary to draw intermediate conclusions and do multiple diagrams. One could also use rules that are similar to those for standard categorical syllogisms. However, both of those methods are based upon the definition of 'some' being 'at least one', so in some cases they will give different results than what my diagrams indicate.

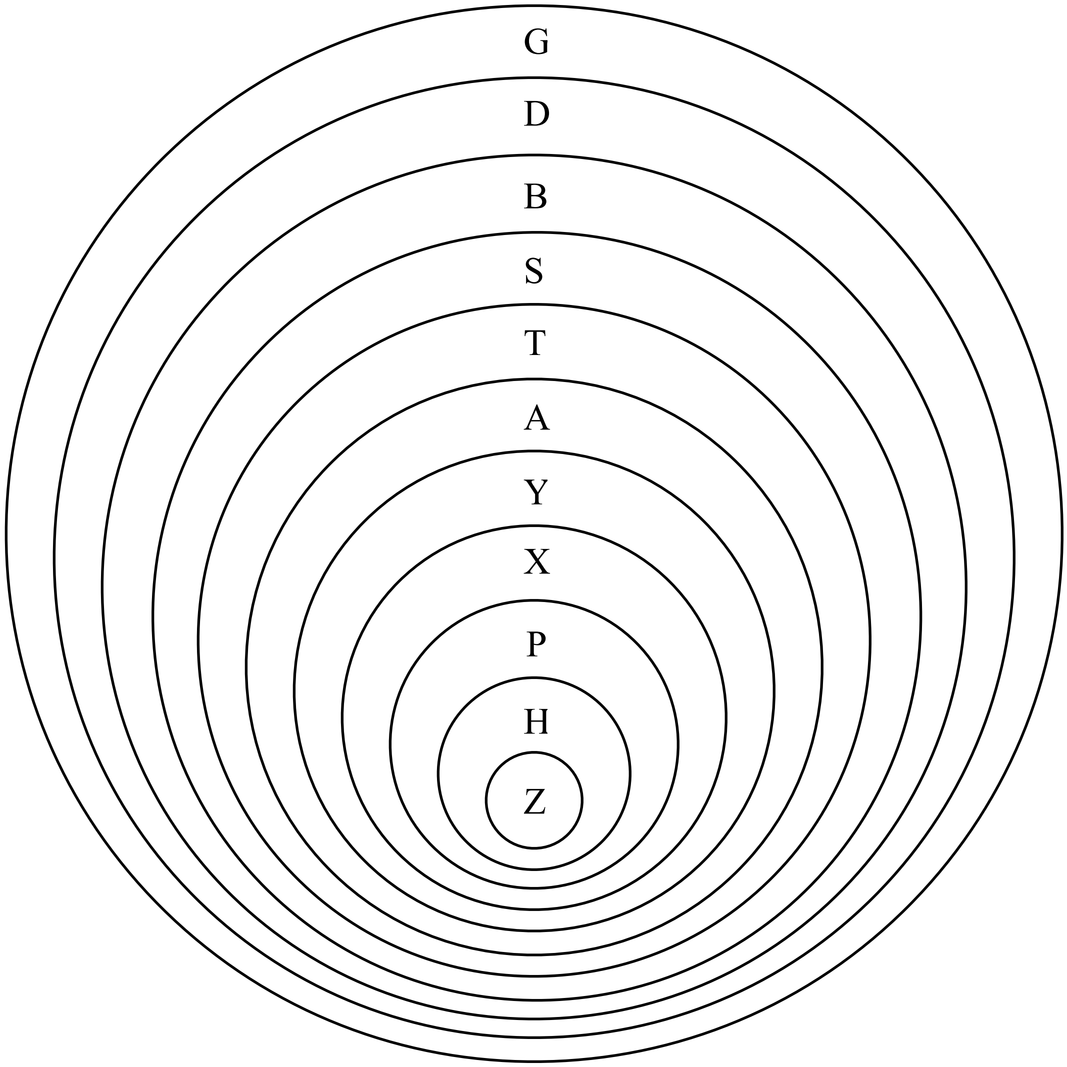
It is possible to diagram a sorites argument without it being in standard form, but it is much easier when it is in standard form. The first premise must contain either the subject term or the predicate term of the conclusion. Since standard form for categorical syllogisms is to have the major premise (the one containing the predicate term) come first, and the minor premise (the one containing the subject term) come second, I think that it makes the most sense to have the major premise come first and minor premise come last in the sorites. All premises should be linked together in a chain by having a term in common with the premise above and/or below it. This means that all of the terms except the subject term and predicate term are middle terms for the sorites. Here is an example, along with the diagram:

All D are G
All B are D
All S are B
<u>All T are S</u>
All T are G



As can be seen from the diagram this argument is clearly valid. It is basically just an extended version of the AAA-1 or 'Barbara' form. One will notice a pattern in the premises where the subject term of the prior premise becomes the predicate term in the next one. There is no limit to the number of premises. As long as the argument has this form it will be valid. Here is one with ten premises:

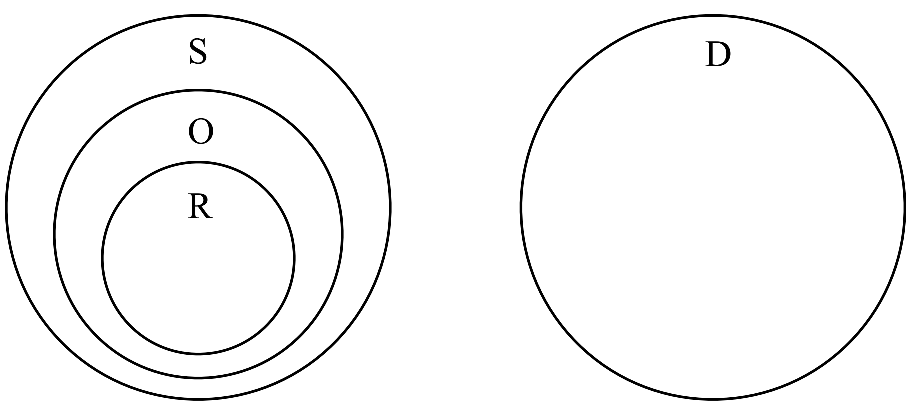
All D are G
All B are D
All S are B
All T are S
All A are T
All Y are A
All X are Y
All P are X
All H are P
<u>All Z are H</u>
All Z are G



The argument would also be valid if the conclusion said 'Some G are Z' because Z is a subclass of G, but that would be equivalent to 'Some P are S', so that is not a standard conclusion.

Now here is one with a negative premise:

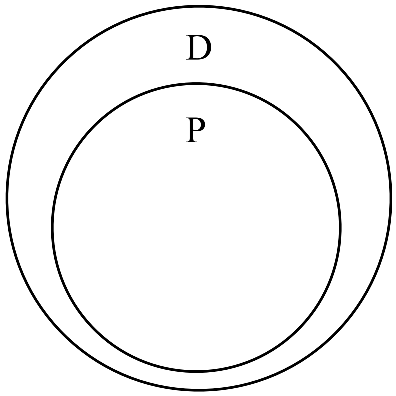
No S are D
All O are S
<u>All R are O</u>
No R are D

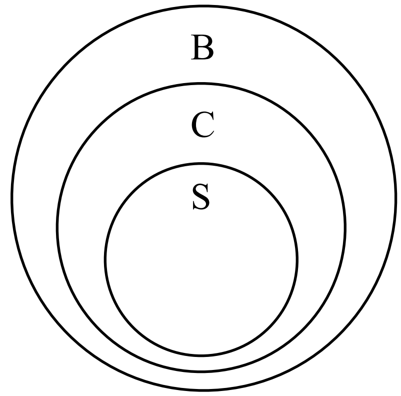


This argument is valid. It is like an extended version of the CAC form.

Here is another:

All P are D
No B are D
All C are B
<u>All S are C</u>
No S are P

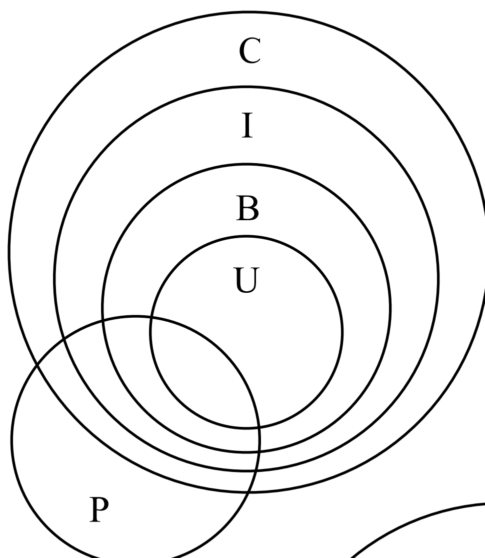


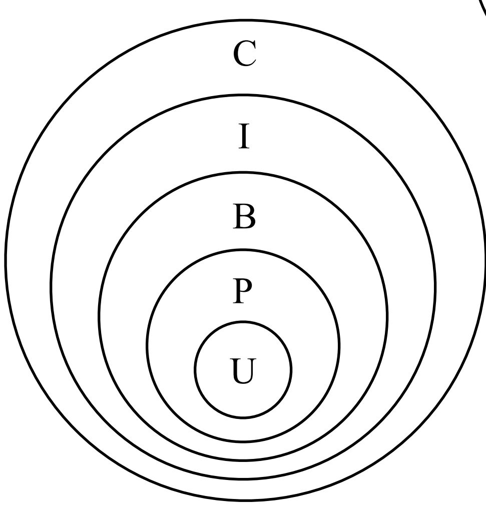


This argument is also valid.

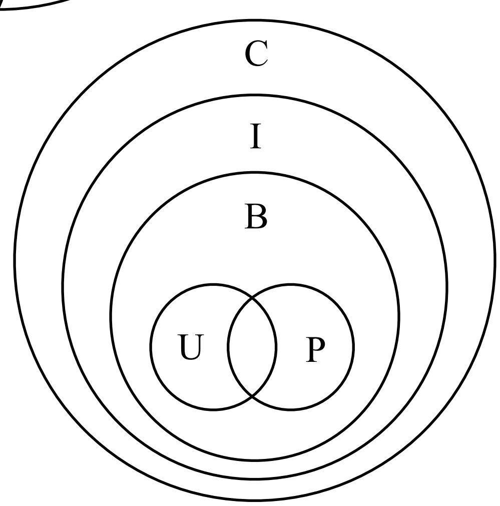
When there is a particular premise it is a little more complicated. We have to consider both ways of diagramming the premise as well as any possible variations on those two ways. For example:

All I are C
All B are I
All U are B
<u>Some P are U</u>
Some P are C





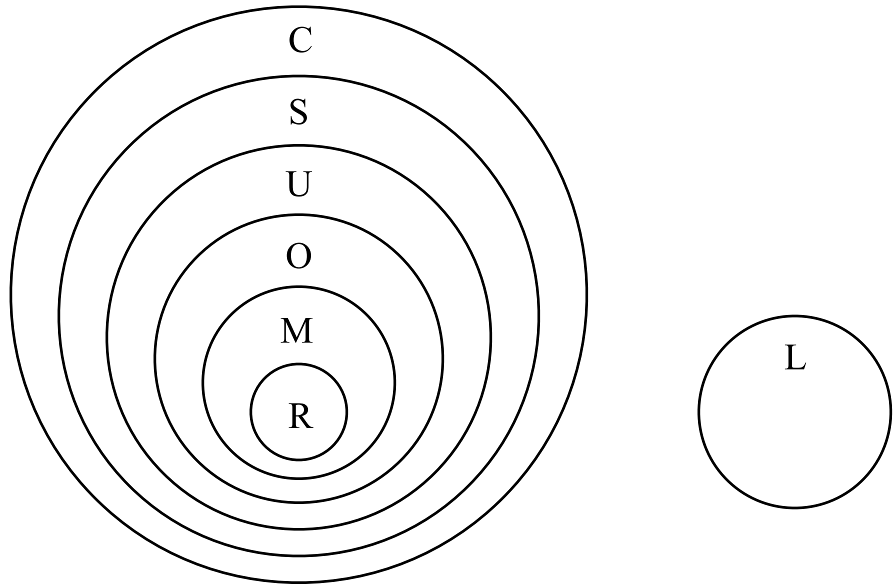
or



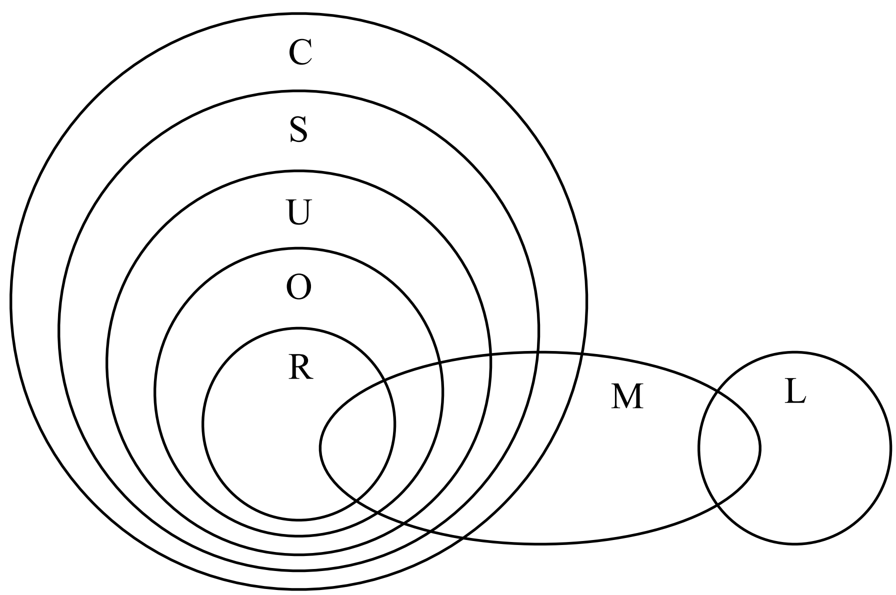
One of the diagrams shows ‘Some P are C’, but in the other two ‘All P are C’ follows from the premises. The conclusion could not be ‘No P are C’, so ‘Either All P are C or Some P are C’ would follow. But because it is possible to get something other than ‘Some P are C’ the argument is invalid. It should be noted that for Aristotle’s interpretation and the ‘Boolean’ interpretation the argument is valid.

Here is another example:

No C are L
All S are C
All U are S
All O are U
All R are O
<u>Some M are R</u>
Some M are L



or

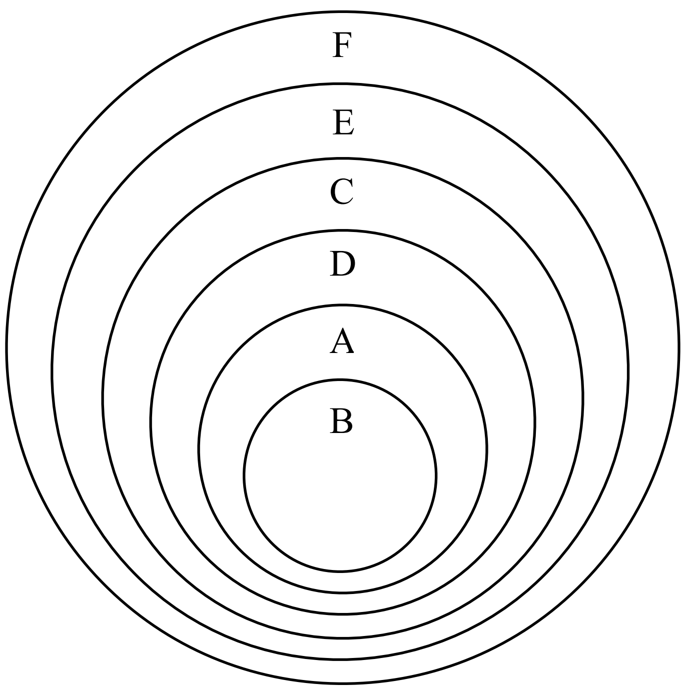


Several other diagrams are possible. For example, R and M could be overlapping inside of O, so that both are subclasses of O, or M could extend beyond C but not overlap L, etc. It could even be the case that L is a subclass of M as long as L does not overlap C. But these two diagrams are enough to show that while it is possible that M overlaps L, the argument is not valid because it does not have to be the case that some M are L. The first diagram shows that it could be 'No M are L'. The only one that it could not be is 'All' because some M are R, and R is a subclass of C, and L cannot overlap C, so there is no way that all M could be L. This means that 'Either Some M are L or No M are L' does follow from the premises, and if this was the conclusion the argument would be valid.

This is another instance where my view differs from the other interpretations. Venn diagrams show this argument would be valid if the conclusion was 'Some M are not L' because according to that view it is possible for 'some are not' to actually be 'none'.

Here is another:

All E are F
All C are E
All D are C
All B are D
<u>Some A are B</u>
Some A are F



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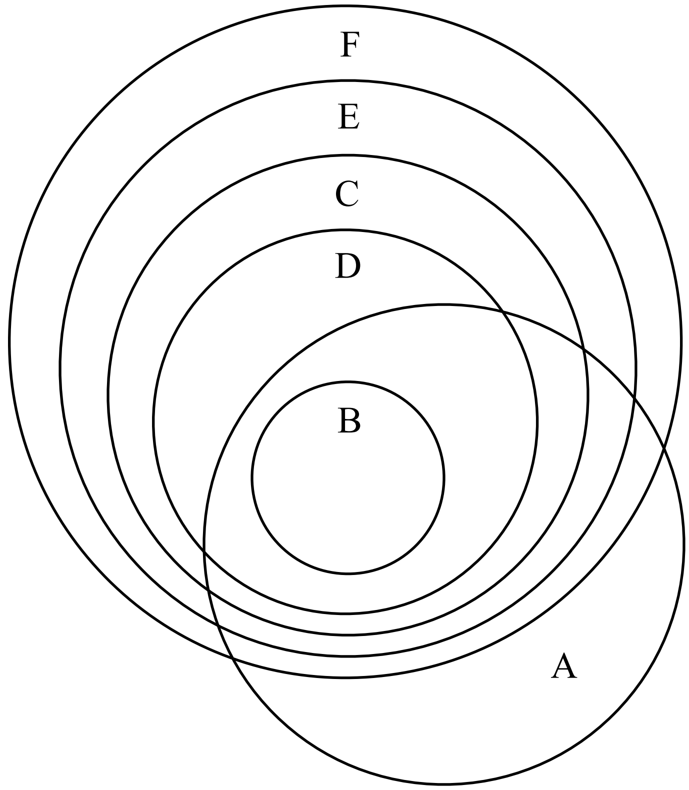

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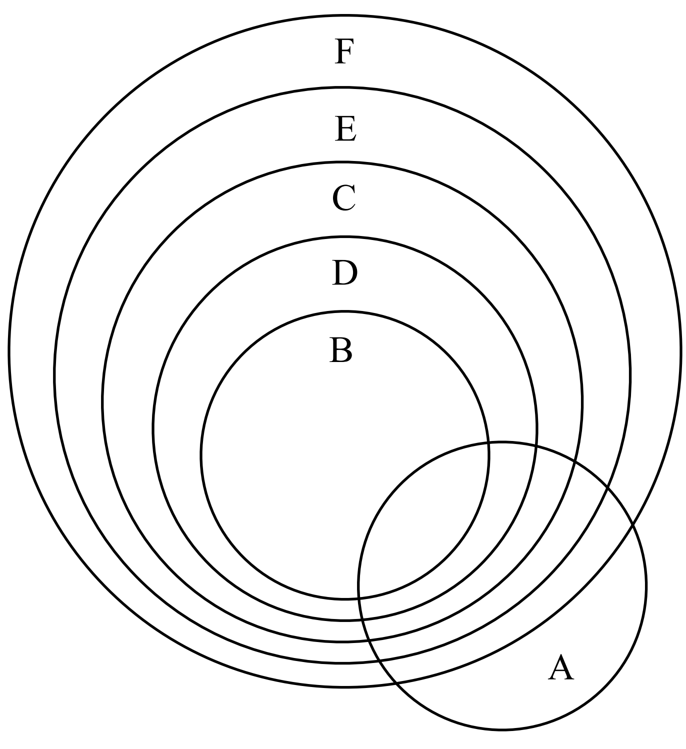
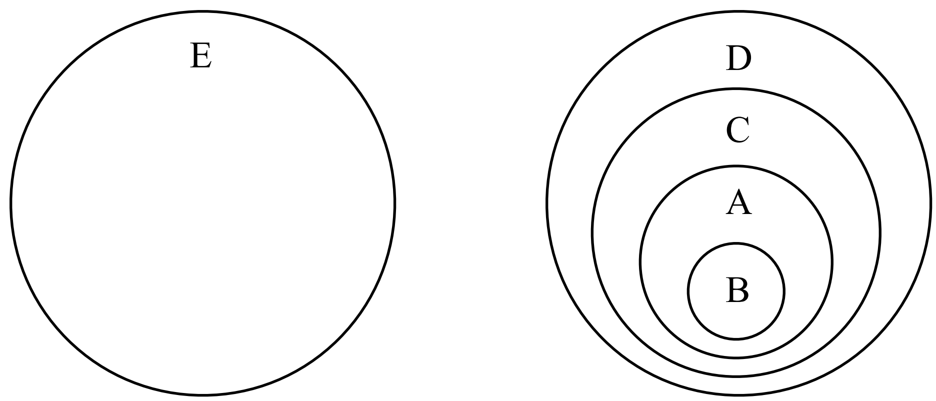


This time ‘No A are F’ can be eliminated as a possibility because B is a subclass of F, and it is also either a subclass of A or overlaps A. However, the argument is not valid because the conclusion does not have to be ‘Some A are F’; if the conclusion was ‘Either Some A are F or All A are F’ it would be valid. For the ‘Boolean’ interpretation, and Aristotle’s, the argument is valid with ‘Some A are F’ as the conclusion.

Let’s do one more. Suppose that we had the following argument:

No E are D
All C are D
All B are C
Some A are B
Some A are not E



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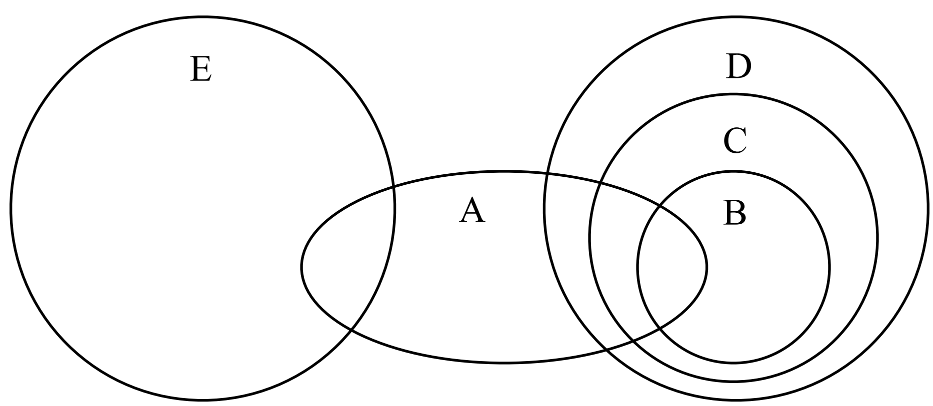


Other diagrams are possible with these premises; for example, A and B could be overlapping classes within C, or A could extend beyond D but not overlap E, B could be a subclass of A but A extends beyond D and overlaps E, or perhaps in another case it does not overlap E, etc. But these two diagrams show the two possible conclusions. It could not be the case that ‘All A are E’ is true. The conclusion could be restated as ‘Some A are E and Some A are not E’ which could be true, as it is in the second diagram, but it could also be ‘No A are E’, as it is in the first diagram. Thus, the argument is invalid. If the conclusion was ‘Either Some A are E or No A are E’ the argument would be valid. Once again this differs from the results given by Venn diagrams and the rules method, both of which would show the argument to be valid as stated.

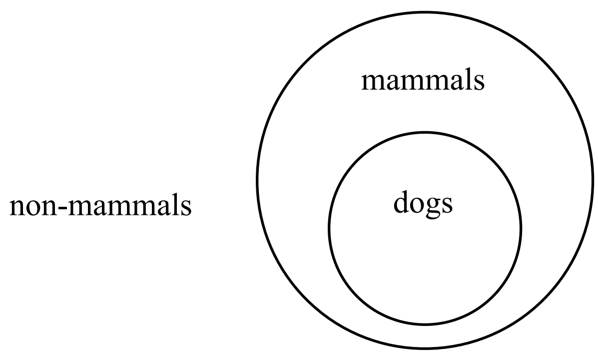
Conversion, Obversion, and Contraposition

This alternative definition of ‘some’ also leads to a few changes to the logical relationships for conversion, obversion, and contraposition.

Original	Converse	Obverse	Contrapositive
A: All S are P	All P are S	No S are non-P	All non-P are non-S
B: Some S are P	Some P are S	Some S are not non-P	Some non-P are non-S
C: No S are P	No P are S	All S are non-P	No non-P are non-S

For A, the converse is not logically equivalent. The contrapositive of A is considered logically equivalent to the original for other views, but as previously discussed, I believe that to be logically equivalent it has to have a roughly equivalent meaning, not just an equivalent truth value. ‘All non-P are non-S’ is the reciprocal or inverse of ‘All S are P’, similar to how ‘Some S are P’ and ‘Some S are not P’ are reciprocals, or ‘A v B’ and ~A v ~B’. Inverse statements are not equivalent in meaning. The A proposition says that every member of S is also a member of P. The contrapositive says that everything which is not a member of P is not a member of S. That will have the same truth value but it does not mean the same thing. The claim is about different members in the second than it is in the first.

Suppose our A proposition is ‘All dogs are mammals’.



This claim refers to members of the dogs class and the claim being made about them is that they are all mammals, or that all of them are also members of the mammals class. The contrapositive of this is ‘All non-mammals are non-dogs’. The contrapositive statement refers to entirely different objects, namely, every object that is not a mammal, and the claim being made about those objects is that all of them are non-dogs, or, by obversion, we could also say that none of them are dogs. None of those objects have ‘dog’ as a predicate, or are members of the ‘dogs’ class. That is not the same claim at all. They will have the same truth value because both are based upon the underlying truth that ‘dogs’ is a subclass of ‘mammals’, but they are not equivalent statements, they are inverted statements, similar to reciprocal fractions.

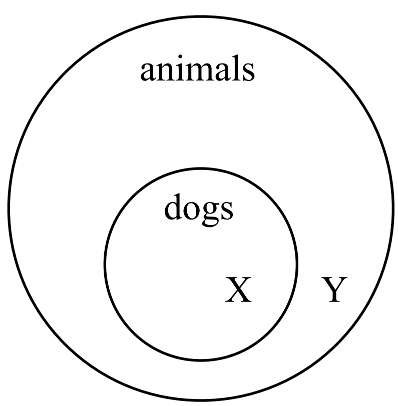
For C, it is the converse that is the inverted claim. ‘No S are P’ refers to the members of S and the claim being made about them is that none of them are P; ‘No P are S’ refers to the members of P and the claim being made about them is that none of them are S. The two statements will have the same truth value but they do not have the same meaning. The contrapositive of C is not equivalent to, or the inverse of the original, it is just an entirely different claim that does not necessarily have the same truth value.

For B, the converse is not logically equivalent. It may seem as though it should be, since conversion is valid for the I statement in Aristotle’s view and the ‘Boolean’ interpretation, but the reason that it is not is because it may be the case that all P are S. For example, it is true that some animals are cats and some animals are not cats, but false that some cats are animals and some cats are not animals.

The contrapositive of B is not equivalent either. This is a little bit harder to see, but examples can be given. We have to consider both types of particular statements, those in which P is a subcategory of S, and those in which they are merely overlapping categories. We will consider the former first. ‘Some animals are dogs’ is true. Is it true that ‘All non-dogs are non-animals’? Certainly not. There are many examples of non-dogs that are animals, such as cats, cows, horses, humans, etc. But ‘No non-dogs are non-animals’ is also false because there are many non-dogs that are non-animals, such as tables, chairs, pens, etc. The proposition that is true is ‘Some but not all non-dogs are non-animals’. So for this example it has an equivalent truth value by default because if the contrapositive of B were not true when B is true then it would have to be the case that either ‘All non-P are non-S’ is true, or that ‘No non-P are non-S’ is true, but neither one of those are true in this case. So far so good; when P is a subclass of S the contrapositive has the same truth value.

However, here we ought to pause and consider carefully exactly what this means. Does this mean that they are logically equivalent statements? Sometimes statements can have the same truth value without being logically equivalent. Let’s use an X and a Y to designate where two specific members are located for ‘Some animals are dogs and some animals are not dogs’:

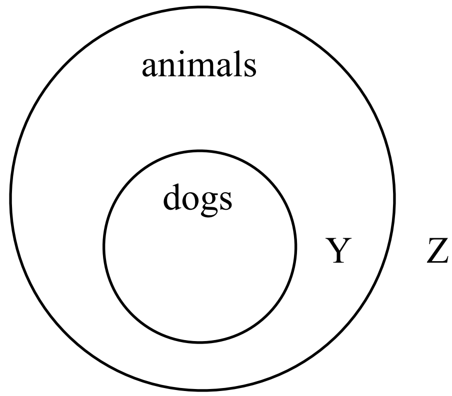
X = King
Y = Vinegar



X represents a specific animal that is a dog, which we will call ‘King’ after my favorite dog, a German Shepherd that I had while growing up. Y represents an animal that is not a dog, let’s have it be a horse named ‘Vinegar’.

Now consider where the members referred to are located for ‘Some non-dogs are non-animals and some non-dogs are not non-animals’:

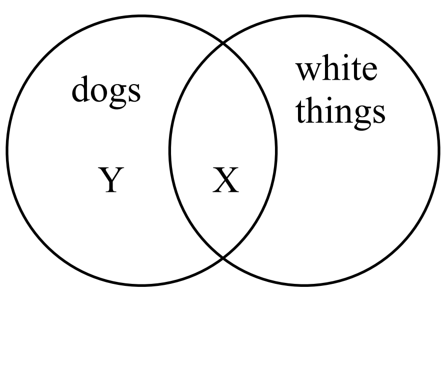
Z = Specific Chair
Y = ‘Vinegar’ (horse)



The non-dog that is not a non-animal, or in other words the non-dog that is an animal could refer to the horse ‘Vinegar’ and/or anything else that is a member of the animals class but not a member of the dogs class, so the ‘O’ part of the particular statement is equivalent. But notice that this second claim does not refer to X, or ‘King’, at all; instead, the first part of it refers to something else entirely. This could stand for anything that is not part of the dogs class or the animals class. Let’s say it is a specific chair which we will call ‘Z’. Obviously the contrapositive is not making the same claim as the original at all. The contrapositive is true because some of the things that are outside of the dogs class are animals and some of them are not animals, but that does not make it an equivalent claim to the original. This statement is not even about dogs at all.

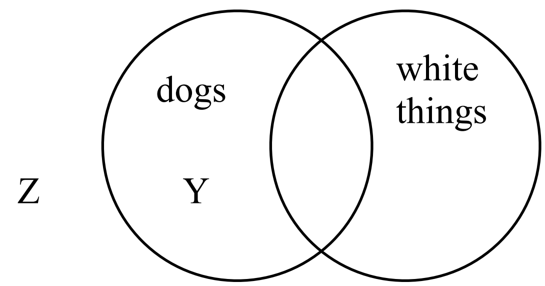
Now we need to consider an example in which the subject and predicate are overlapping categories. ‘Some dogs are white’ is true. Would it be true that ‘All non-white things are non-dogs’? Clearly not, because dogs come in many different colors that are not white, such as brown, black, red, etc. This is saying that if it is a non-white thing then it cannot be a dog, and that is obviously false. But of course it would also be false to say ‘No non-white things are non-dogs’. Think about all the non-white objects (meaning an object of any other color) that would not be dogs! There would be a huge number of them, such as red shoes, a blue bike, a yellow car, etc. Once again, the categorical proposition that is true along with ‘Some dogs are white’ is ‘Some non-white things are non-dogs’. But are these statements really saying the same thing?

Here is where two representative members would be located for the first statement:



X = dog that is white
Y = dog that is not white

Here is how the contrapositive would look:



Z = non-white thing that is a non-dog
Y = non-white thing that is a dog

Here we see something similar to the prior example. The O portion of the statement is equivalent, but the I portion is not. Thus, ‘Some dogs are white and some dogs are not white’ is not equivalent to ‘Some non-white things are non-dogs and some non-white things are not non-dogs’.

We have still only considered instances in which B is true. When B is false, such as ‘Some cats are mammals’ (because all cats are mammals) the contrapositive ‘Some non-mammals are non-cats’ is false as well because in reality all non-mammals are non-cats. It must be so because cats is a subcategory of mammals.

But here is where we can find a counterexample that shows they do not always have the same truth value: ‘Some cats are fish’ is false, but ‘Some non-fish are non-cats’ is true. Dogs, humans, and tables would all be examples of things that are non-fish and also non-cats. Could it be the case that all non-fish are non-cats? No, a cat is not a fish, so cats would be an example of a non-fish that is a cat. Therefore we have found a case in which B is false and its contrapositive is true.

These results are not all that surprising because in other versions (both Aristotle’s and the ‘Boolean interpretation’) I is not logically equivalent for contraposition but O is; for conversion, I is equivalent but O is not. For my version, if either I or O is not equivalent then the particular statement as a whole is not equivalent.

Although the converse of B is not equivalent to B, if you were told, or knew based upon the categories referred to that S and P were overlapping categories rather than P being a subcategory of S, then conversion would be legitimate in that specific instance.

Also, though conversion for the A statement is not equivalent, conversion by limitation is valid. If all dogs are mammals then it must be the case that some but not all mammals are dogs. The ‘Boolean’ or Venn interpretation does not recognize this relationship because of existential import, but I believe that it should be considered valid. If the subject or predicate is hypothetical, then the implication is only hypothetical, but the inference is still valid. It is an alternative way of identifying the same relationship, which is that S is a subcategory of P. It would obviously not be valid going the other direction; if all that we know is ‘Some but not all P are S’ we could not necessarily conclude ‘All S are P’ because it is possible that S and P could be merely overlapping categories. But, if ‘All S are P’ is true then you know that S is a subclass of P, so ‘Some P are S’ must also be true.

One of Venn’s arguments for his view of existential import was that if the universal affirmative statement was considered to have existential import then its contrapositive would also need to have existential import, and he felt that this led to absurdities. I do not think that is the case at all because the claims refer to different objects. If one of the categories did not have any members then the relation would only hold hypothetically. For example, ‘All men are mortal’ would ordinarily have the same truth value as ‘All non-mortals are non-men’ but the second may not have an actual truth value because there may be no such thing as an immortal or ‘non-mortal’ (meaning something that goes on forever and is not subject to decay). We could only discover whether there is such a thing empirically, not through a logical deduction. Such a deduction does not force one of the classes to have members. If in fact there are no ‘non-mortals’ then the contrapositive is only hypothetically true.

It is true hypothetically and in the actual world that ‘No humans are hobbits’ because humans exist in both the *Lord of the Rings* universe and in the actual world, but the converse statement ‘No hobbits are humans’, while true for Middle earth, is undefined for the actual world. One could also say that it is hypothetically true, meaning that if there were hobbits in the actual world none of them would be humans. In the vast majority of cases this is not an issue at all because both the subject and the predicate classes will have actual members. But in the rare instances in which it does come up, the operations result in statements that are only hypothetical or only apply to a fictional setting.

The one operation we have not yet discussed is obversion. The obverse has the same truth value as the original statement for all three categorical propositions. However, it is not equivalent in meaning. ‘A and B’ is equivalent to ‘B and A’, both logically, and in meaning, but obversion is similar to inversion in that it has the same truth value as the original but it is actually saying something that is the opposite of the original. For obversion, the obverted statement refers to the same subject class but it is the predicate and the quantifier that are inverted. The obverse of A is ‘No S are non-P’. It has the same truth value as ‘All S are P’ but it is obviously making a different claim about S. It is a different, and in fact exact opposite predicate, and the quantifier is the opposite of the original. ‘All S are non-P’ is the obverse of ‘No S are P’ and it does the same thing. ‘Some S are not non-P’ is unique in that it appears to be the case that the quantifier is not inverted, but actually it is because the copula is inverted. The quantifier goes from ‘Some’ in the original to ‘Some are not’ in the obverse. Once again the predicate is the opposite of the original predicate. If some S are not non-P then those S are P, so if the obverse is true the original must be as well. And if there is an S which is a P then it is an S that is not a non-P. The obverse is the opposite way of stating the claim.

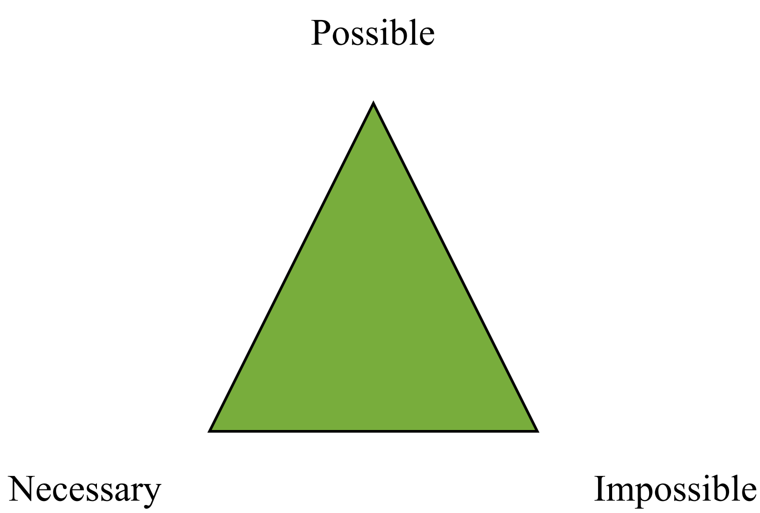
The way that inversion differs from obversion is that an inverse claim is not even about the same subject as the original statement. ‘Some S are not P’ does refer to S, but a different part of the S class than the S referred to in 'Some S are P'. Obversion and inversion are similar in that both make an assertion that is opposite in some way to the assertion that is made in the original statement and because of that symmetry they both have the same truth value as the original. One is like a mirror image that differs from the original in that what was on the left side is now on the right, and the other is kind of like a reflection that is upside down.

To summarize:

Original	Obverse	Inverse
A: All S are P	No S are non-P	All non-P are non-S
B: Some S are P	Some S are not non-P	Some S are not P
C: No S are P	All S are non-P	No P are S

The Modal Triangle

Aristotle's categorical logic was tied to his metaphysics. During the Middle Ages logicians felt that it would be best to keep them separate and drew a sharper distinction between them. It is true that we should not conflate accidental and necessary predication with universal and particular claims. It is possible to have a universal claim that is merely accidental predication. For example, someone with a team of sled dogs might say 'All of the dogs are sleeping'. The category being referred to is the sled dog team, not all dogs in the world. It is a universal claim because it refers to every member of the category but of course being asleep or awake is not an essential property of those dogs even if all of them happen to have it at that particular time. In most cases 'All' would imply necessary predication, and 'Some' accidental predication, but there could be instances in which that is not the case. For these reasons it probably is best to keep metaphysics out of it, and because of that, it is probably best to keep categorical and modal logic separate. Nevertheless, there is an undeniable correspondence. One could easily construct a Modal Triangle of Opposition:



Something that holds for all possible worlds is necessary, something that holds in no world is impossible, and something that holds in some worlds is possible (or contingent). In other words for anything that is possible there is at least one possible world in which that state of affairs is realized and at least one other world in which it is not.

Sometimes people say that something is necessary *in* all possible worlds, but I prefer to say instead that it holds *for* all worlds so that it does not make it seem as though the subject has to be instantiated in all worlds. A bachelor is unmarried for all possible worlds because in every world in which the subject is instantiated he never has the predicate of being married, but of course there are possible worlds in which there are no bachelors at all. The necessity is not that there is a subject, it is that if there is a subject it always has or lacks a certain predicate (and simultaneously has or lacks the antipredicate, or predicate complement, in this case 'non-married'). A form of obversion applies here as well: you could either say that for all worlds a bachelor is non-married or that there is no world in which a bachelor is married.

To say that a thing is possible means that the subject may or may not have that predicate without contradiction, so in some worlds it would have it and in others it would not.

If a thing is impossible the subject could not have that predicate because it would result in a contradiction if it did. There would be many worlds with squares, and many with circles, and some that have both, but there would be no square-circles in any possible world, as that combination of subject and predicate is impossible. (As are circular-squares.)

But suppose there was something that is defined as being instantiated in all worlds: does that mean that it really is in all worlds?

Consider the following argument:

1. If there is such a thing as a being whose existence is necessary then by definition it would be instantiated in all possible worlds.
2. There is a possible world in which there is a being whose existence is necessary.

Therefore, a being whose existence is necessary is instantiated in all possible worlds.

The claim is that if a necessary being is even possible it must be instantiated in all worlds, including the actual one. Unless you can demonstrate that a necessary being is impossible you would have to grant the premise that there is at least one possible world in which it is instantiated, or so it is believed. Based upon how a necessary being is defined in the first premise, the conclusion validly follows from the two premises.

However, the argument is unsound because premise 2 is actually self-contradictory. Just as in the triangle for categorical logic, all three modal claims are mutually exclusive. Only one is true. No one would try to say that something is both possible and impossible at the same time, but many try to do that very thing with possibility and necessity. Something which is possible does *not* hold for all worlds; if it did, it would be necessary. It has to be either possible or necessary, it cannot be both at the same time. So you cannot say that there is one possible world (or a possible world) in which it is instantiated without implying that its instantiation is possible, which would mean that there is at least one other world in which it is not instantiated, an obvious self-contradiction. Even in Aristotelian categorical logic you cannot go from the truth of the particular I statement to the truth of the universal A statement, which is similar to what this argument is trying to do. To be self-consistent the premise would have to say 'In all possible worlds there is a being whose instantiation is necessary' because being instantiated in all worlds is how 'necessary being' is being defined. But there is no reason to think that such a premise is true. In fact, because it does not result in a contradiction to imagine a possible state of affairs in which there is no such thing as a necessary being there is good reason to reject it.

So how should one interpret the statement: 'I guess it is possible that there could be a necessary being.' I do not think it would be correct to interpret it to mean that there is a possible world in which it is so. What it really means is that the speaker thinks that the claim 'There is a necessary being instantiated in all possible worlds' could be true, but it also might not be, they don't know for sure. This is a type of second-order possibility, which I call meta-possibility, that is not represented in standard modal logic. Claims that are about all possible worlds are meta claims, which themselves would be necessary, possible, or impossible, but this must be represented in meta-possible worlds rather than the standard possible worlds. The claim should be interpreted to mean that in at least one meta-possible world (but not all of them) there is a being who is instantiated in all standard possible worlds, which would mean that its instantiation is necessary relative to those worlds. But since there are other meta-possible worlds in which that is not the case, we would not know whether a necessary being really is instantiated in all possible worlds or not.

Now perhaps you will object that a necessary being would have to be instantiated in all meta-possible worlds as well. Maybe it would if there really is such a thing, but that is the problem, we do not know for sure whether is such a thing. Because of that, I certainly would not believe a premise which says that it is necessary that such a being is in all meta-possible worlds any more than a premise which says that it is necessary that it is in all standard possible worlds. What would be the justification to believe that is true? Now of course it does seem possible that a necessary being could be instantiated in all meta-possible worlds, but also possible that it might not be; this would be third-order possibility that is outside of the meta-possible worlds set. It is the same problem, we have just moved up a level. This line of argument could continue on to even higher levels but it would not ever get us any closer to proving that there really is a necessary being.

What has been said so far refers to things, but you could also use the possible worlds framework for statements too. A self-contradictory statement, such as 'A and not-A' is not true in any possible world for much the same reason that a subject cannot have a predicate that would contradict the subject's own definition. Assuming that it is possible for A to be true, and also possible for not-A to be true, there are worlds in which A is true and worlds in which not-A is true, but no world in which they are both true at the same time and place. We can regard a self-contradictory statement as impossible because there is no possible world in which it is true. There are worlds in which it appears, but when it does it is always false. For statements that are possible there would be at least one world in which it is true and at least one other world in which it is false. Statements that are necessary, such as ‘Either it is raining or it is not’, would be true in every possible world in which they are present.

Propositional Logic

The three categorical propositions are translated into propositional logic in the following way:

A: S⟶P Translation: If it is an S then it is a P.
B: (*i*)(S⟶P) ^ (*i*)(S⟶~P) Translation: In some instances if it is an S then it is a P, and in some instances if it is an S then it is not a P.
C: S⟶~P Translation: If it is an S then it is not a P.

These translations are not an exact match to the categorical propositions but they are as close as we can get in propositional logic. Like any language, some things are lost in translation.

The symbolization used for B is unique, so I will explain it further. Boole used the letter v to stand for ‘some’. I like his general idea, but with a few modifications. He symbolized it algebraically, I have chosen to use propositional and predicate logic. The choice of which letter to use is somewhat arbitrary, but it is probably best to use one that is not used frequently elsewhere. I have chosen to use *i* (which stands for ‘instance’) because v could be confused with the wedge symbol for ‘or’, especially when they are right next to each other. A universal quantifier for A and C is not necessary because it is implied that we are talking about all instances of S unless otherwise noted.

With this one alteration we have introduced quantification into propositional logic and you could symbolize many of the arguments that you do in predicate logic but in a much simpler format. That is nice because predicate logic can be quite cumbersome at times.

Here are a few other items of interest:

Hypothetical syllogism has a similar form to AAA-1(aka Barbara). Here is the standard form for hypothetical syllogism:

$p \rightarrow q$
$\underline{q \rightarrow r}$
$p \rightarrow r$

If the premises were in a different order it would look like this:

HS	AAA-1
M→P	All M are P
S→M	All S are M
S→P	All S are P

A hypothetical syllogism can be extended just like AAA-1. For example:

A→B
B→C
C→D
D→E
E→F
F→G
G→H
H→I
I→J
J→K
K→L
L→M
M→N
N→O
O→P
P→Q
Q→R
R→S
S→T
T→U
U→V
V→W
W→X
X→Y
Y→Z
A→Z

Notice that we have the same criss-cross pattern as with the AAA-1 sorites, although it is going in a different direction. In this case the consequent in one premise is the antecedent in the next one. The number of premises that one could have is unlimited. As long as all premises are linked together in this way so that they form a chain, and all of them are true, there is no way that the conclusion could be false. I would consider the premises to be true if the antecedent legitimately guarantees the consequent for each conditional statement and the antecedent condition has been met so that it is actual. If any of the conditions have not been met then the conclusion could be hypothetically true, but not realized in actuality. In a way, each consequent is like a subconclusion, and we have several smaller inferences embedded into the larger one. A conditional is not a proposition, it is an inference from antecedent to consequent; the inference itself does not have a truth value. But if it is meant to function as a premise we could say that the conditional inference is true or correct just as one might say that it is true that an argument is valid. This can be used for conditional statements about class inclusion and exclusion, such as 'If it is a dog then it is a mammal' but it could also be used for other types of conditional statements as well as long as the antecedent entails the consequent for each.

You could also have an extended sorites form of Modus ponens and Modus tollens.

MP
A→B
B→C
C→D
D→E
E→F
F→G
G→H
H→I
I→J
J→K
K→L
L→M
M→N
N→O
O→P
P→Q
Q→R
R→S
S→T
T→U
U→V
V→W
W→X

$X \rightarrow Y$

$Y \rightarrow Z$

$\underline{A\qquad}$

Z

MT

$A \rightarrow B$

$B \rightarrow C$

$C \rightarrow D$

$D \rightarrow E$

$E \rightarrow F$

$F \rightarrow G$

$G \rightarrow H$

$H \rightarrow I$

$I \rightarrow J$

$J \rightarrow K$

$K \rightarrow L$

$L \rightarrow M$

$M \rightarrow N$

$N \rightarrow O$

$O \rightarrow P$

$P \rightarrow Q$

$Q \rightarrow R$

$R \rightarrow S$

$S \rightarrow T$

$T \rightarrow U$

$U \rightarrow V$

$V \rightarrow W$

$W \rightarrow X$

$X \rightarrow Y$

$Y \rightarrow Z$

$\underline{\sim Z\qquad}$

$\sim A$

Predicate Logic

In predicate logic the three categorical propositions should be symbolized this way:

A = Ux(Ax→Bx)
B = Px(Ax→Bx)
C = Ux(Ax→~Bx)

'Ux' stands for universal, meaning that for all x, if x is A then x is B. 'Px' stands for particular. 'Particular' in logic is defined as: denoting a proposition in which something is asserted of some but not all of a class. This definition is perfect for what we want: for some but not all x, if x is A then x is B. The C proposition means that for all x, if x is A then x is not B.

One of the rules of inference used in proofs is simplification. The rule states that if two propositions are given as true on a single line then each of them is true separately. If a conjunctive statement is true then both of its conjuncts have to be true, so it would also be a true statement if we only used one of those conjuncts on a new line. I have been using similar reasoning to say that in most cases we only need to state part of the particular proposition and we can leave the other part implied. It is simply for greater convenience that we often choose to state only one of the conjuncts, but both are true if B is true. Thus, the shorthand version and the full statement can be used interchangeably. (The double colon is used to designate equivalence.)

Px(Ax→Bx) :: Px(Ax→Bx) ^ Px(Ax→~Bx)
Px(Ax→~Bx) :: Px(Ax→Bx) ^ Px(Ax→~Bx)

This is only allowed if the conjuncts of the compound statement are obverse, inverse, or logically equivalent to each other. It works because they always have the same truth value, so if one of them is true the other must be as well, and a compound statement in which they are conjoined would also be true.

Quantifier Negation Rule

In predicate logic the following are considered to be logical equivalences:

(x)(Fx) :: ~(∃x)(~Fx)
~(x)(Fx) :: (∃x)(~Fx)
(∃x)(Fx) :: ~(x)(~Fx)
~(∃x)(Fx) :: (x)(~Fx)

The symbolization used here is different than what I use: ‘x’ or sometimes ‘∀x’ means all, or for any x; ‘∃x’ means there is an x, or an x exists such that . . . The latter is thought to have existential import, while the former does not, which is one reason that I do not use these symbols. The first means ‘everything is F’ which is considered equivalent to ‘it is not the case that there exists an x that is not F’. The basic idea here is that each proposition is equivalent to the negation of its contradictory.

However, as previously noted, for the Triangle of Opposition none of the three propositions are directly contradictory to the others. Instead their contradictories were these:

A = All As are Bs	contradictory	Either No As are Bs or Some As are Bs
B = Some As are Bs	contradictory	Either All As are Bs or No As are Bs
C = No As are Bs	contradictory	Either Some As are Bs or All As are Bs

To have something that is logically equivalent to the original (or at least has the same truth value) we would need to negate the contradictory:

A = Ux(Ax→Bx) :: ~[Ux(Ax→~Bx) v Px(Ax→Bx)]
B = Px(Ax→Bx) :: ~[Ux(Ax→Bx) v Ux(Ax→~Bx)]
C = Ux(Ax→~Bx) :: ~[Px(Ax→Bx) v Ux(Ax→Bx)]

I defined ‘some’ as ‘not all and not none’ and you could do the same thing for the other two quantifiers: ‘all’ could be defined as ‘not some and not none’, and ‘none’ could be defined as ‘not all and not some’. The statements above are just more technical versions of that. This could be shown using DeMorgan’s rule, which says that ~(p v q) is logically equivalent to ~p ^ ~q, if these were ‘inclusive or’ statements; however they are actually ‘exclusive or’, and DeMorgan’s rule does not ordinarily hold for ‘exclusive or’. The reason is that an ‘exclusive or’ could be false if both disjuncts are true as well as if both disjuncts are false, so ~(p v q) could mean that p ^ q is true just as easily as ~p ^ ~q. But ‘Some S are P’ could not be interpreted to mean ‘Both All S are P and No S are P’ because that would be self-contradictory; if ‘Some S are P’ was equivalent to that it would always be false. Therefore we can eliminate that as a possibility in this case and say that ~(p v q) is equivalent to ~p ^ ~q.

For the claim above concerning F here is what would be equivalent (at least somewhat) and what is contradictory:

Ux(Fx) :: ~[Ux(~Fx) v Px(Fx)]	contradictory	~Ux(Fx) :: Ux(~Fx) v Px(Fx)
Px(Fx) :: ~[Ux(Fx) v Ux(~Fx)]	contradictory	~Px(Fx) :: Ux(Fx) v Ux(~Fx)
Ux(~Fx) :: ~[Ux(Fx) v Px(Fx)]	contradictory	~Ux(~Fx) :: Ux(Fx) v Px(Fx)

It is important to translate propositions accurately. ‘It is not the case that there are any werewolves’ or ‘There is no such thing as a werewolf’ or ‘Werewolves do not exist’ are universal statements because they apply to all members of the class; all three should be translated as Ux(~Wx), or ‘for any x, x is not a werewolf’ (or ‘x is a non-werewolf’). It should not be symbolized as ~Px(Wx), or ‘it is not the case that for some x, x is a werewolf’. I think many would be inclined to translate it that way because they are so used to the particular being tied to existence with the ‘existential quantifier’ but negating the particular implies that either Ux(Wx) is true, or Ux(~Wx) is true. That is not really an accurate representation of the claim that these propositions are making: they only want to express the latter. It also could not be Px(~Wx) because that would simultaneously imply Px(Wx), or for some x, x is a werewolf, which is the opposite of what the original statements were saying. Another possibility would be to symbolized them as ~Ux(Wx), meaning ‘it is not the case that everything is a werewolf’ or ‘not everything is a werewolf’, which of course is true, but not specific enough: it could mean either Px(Wx) is true or Ux(~Wx) is true. It is obvious that the original statements in English were only meant to express the latter. Once again Ux(~Wx) is the one that is true, and this captures the meaning of the statements the best. (This does have a truth value because the claim is about the subject’s existence; if the claim was that the subject had or lacked some other predicate it would be undefined.)

For any ordinary language categorical statement we really should be able to translate it into one of the three standard forms. Translating it in a way that negates the quantifier just introduces unnecessary complexity and ambiguity, and should be avoided.

Finite Universe Method

The underlying idea of the finite universe method is that a valid argument remains valid no matter how things in the actual universe might be altered. I am not entirely sure that is correct, but maybe it is. At any rate, the method presupposes that a valid argument remains valid even if it was the case that the universe contained only one member. If that were true, according to this view, the meaning of universal and particular statements would be altered.

Suppose that it is the case that there is nothing outside of the universe, and that it contained only one member. We will call that one thing ‘Anna’. The statement ‘Everything is beautiful’ would then be equivalent to ‘Anna is beautiful’ because Anna is everything. According to this view, ‘Something is beautiful’ would also be equivalent to ‘Anna is beautiful’ because Anna is that something. Thus, ‘Everything is beautiful’ would be equivalent to ‘Something is beautiful’.

However, as previously discussed, a particular statement cannot be true if there is only one member of the subject category, and it would be the same way if there was only one thing in the universe. I have the feeling that a subtle shift in meaning is taking place here with the word ‘something’: it could, and often does mean ‘some but not all things’ but it could also just mean ‘a thing’. If the latter is the intended meaning then perhaps you could interpret ‘Something is beautiful’ to mean ‘a thing is beautiful’ or ‘the thing is beautiful’, but those are universal statements if that thing is all there is - it must either have the predicate or not.

‘Something is beautiful’ would only be equivalent to a particular categorical statement if it meant ‘Something is beautiful and something is not beautiful’. But that would contradict ‘Everything is beautiful’ and would only be true if another thing exists or that one thing is both beautiful and not beautiful at the same time. ‘Some’ is never equivalent to ‘all’ or ‘none’ under any conditions, including when the universe (or the subject category) has one member.

If we were to assume that the universe contains two things, let us call them ‘Anna’ and ‘Brandy’, the finite universe method assumes that the statement ‘Everything is beautiful’ is equivalent to ‘Anna is beautiful and Brandy is beautiful’ while ‘Something is beautiful’ would be equivalent to ‘Anna is beautiful or Brandy is beautiful’. The disjunction is thought to be an ‘inclusive or’ meaning that it would be true if both Anna and Brandy are beautiful, and in fact that is the only way that it could be true at the same time as ‘Anna is beautiful and Brandy is beautiful’. But to accurately represent the particular categorical statement it would have to be translated as an ‘exclusive or’ which is only true when one of the disjuncts is true and the other is false. This is because if there are only two things in the universe (or only two members of any category), and the particular statement is true, then it would have to be the case that one member has the predicate and the other member does not. Thus, the particular statement is true if Anna is beautiful and Brandy is not, or if Brandy is beautiful and Anna is not, but it is false if both are beautiful. Obviously this is not consistent with ‘Anna is beautiful and Brandy is beautiful’.

If there were three members, let’s call the third Christine, then ‘Everything is beautiful’ would be equivalent to ‘Anna is beautiful, and Brandy is beautiful, and Christine is beautiful’. ‘Something is beautiful’ would be equivalent to ‘Anna is beautiful, or Brandy is beautiful, or Christine is beautiful’. At least one of these would have to be an ‘exclusive or’ to accurately represent the particular (‘Something is beautiful and something is not beautiful’) because at least one member must have the predicate and at least one other member must not. Without more information we would not know whether only one of the three had the predicate, or whether two of them had it and one did not, or which, but there has to be at least one of each, which means that this is not consistent with the ‘and’ statement. That would be the case no matter how many members of the universe there are.

Definite Descriptions

A definite description, as the name implies, describes an individual person, place, or thing. An example would be ‘the first president of the United States’, which, of course, describes George Washington. Definite descriptions are like names in that they identify a particular thing, but they do it by describing a situation or relationship that only that one thing satisfies. Bertrand Russell believed that definite descriptions assert three things: the thing being described exists, there is only one thing that fits the description, and it really does have the attribute assigned to it by the statement. Thus, Russell believed that definite descriptions have existential import. Suppose that we had a statement like the following: ‘The queen of the United States is a woman.’ Russell would say that it is false because no such person exists, or has ever existed. Most logicians today accept this interpretation, which is strange to me. It is inconsistent to say that this statement has existential import but ‘All things identical to the queen of the United States are women’ does not. Both claims refer to the exact same subject, and they are even making the exact same claim about that subject, they just do it in a slightly different way. What is the justification to say that one is true and the other is false?

As for my own view, I would have a similar answer here as what I have said previously. If the description is of something that does not actually exist (and never has, so that it does not describe something that is actual) then it is not true or false, it is undefined. But it could have a hypothetical truth value. The description above is hypothetically true because all queens are female, by definition, so assuming that we have restricted the universe of discourse to humans, if there was a queen of the United States she would be a woman. That much is *a priori* - there does not need to be an actually existing subject to know that would be true, it is just not realized in the actual world.

‘The faithful but gullible squire of Don Quixote is Sancho Panza.’ This does not describe a person place or thing in the actual world, so it is undefined for the actual world, but it should be considered true relative to fiction. If the same description was used for Achilles instead, it would then be false (relative to the context of fiction) because it would not be an accurate description of the character.

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