An Alternative Version of Categorical Logic (Abridged)

by David Johnson

The ‘Some’ Problem

Suppose that you heard someone say the following: ‘It was a great party. Everybody was really nice. Some of them even stayed after and helped us clean up.’ Most people would interpret this to mean that not everybody who went to the party stayed after to help clean. But not a trained logician. He would gleefully point out that ‘Some S are P’ does not necessarily imply ‘Some S are not P’. He would give as a counterexample something like the following: ‘If it is true that some dogs are mammals (and technically it is, since ‘some’ is defined as ‘at least one’) does that mean that some dogs are not mammals?’ Obviously it would not, because as everyone knows, it is actually the case that all dogs are mammals. So, even though the speaker said ‘some’, for all we know it could have been everybody.

Good thing the party is over, because if it was not, this is where it would take a turn for the worse, as I would have to take issue with the logician. I would ask him why he thought that the speaker would use the word ‘some’ if what she really meant was ‘all’. If she knew that everybody stayed to clean why wouldn’t she just say that?

It seems to me that the logician is not playing by the rules. He is using terms in a different way than how they are used and understood outside of logic. In standard everyday discourse people do not ever use the term ‘some’ to mean ‘all’, so for them ‘some’ does imply ‘some are not’. As far as his counterexample, I would ask why anybody would want to make the claim ‘Some dogs are mammals’ and whether that is actually even true. It is true that at least one dog is a mammal, I’ll grant him that, but is ‘at least one’ really the correct definition of ‘some’?

Imagine that you were on a ship and when you saw some humpback whales another tourist nudged you and said: ‘Did you know that some whales are actually mammals?’ Wouldn’t you feel the need to gently correct him and say: ‘Uh, well actually all whales are mammals.’ Don’t you think that somebody else would probably say it if you didn’t? I think it would actually be very odd if everybody just agreed with him. I mean, at the very least, his statement seems imprecise. Why not just say ‘all’ if it is actually all? It is kind of like saying that in basketball the team that scores the most points usually wins, or sometimes two plus two equals four. And speaking of the word ‘sometimes’, we never use that to mean ‘all times’ now do we?

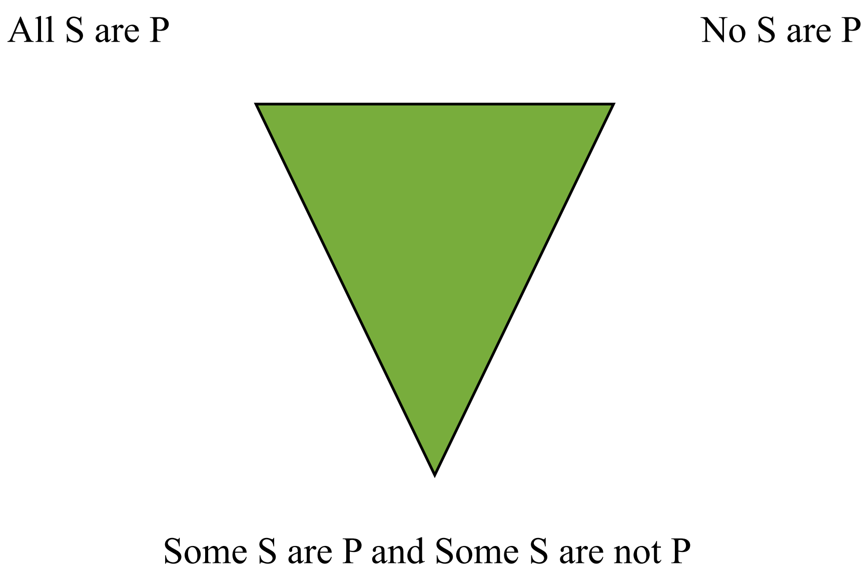
The way that the logician uses the word, ‘some’ could also mean none. This happens with negative claims. ‘Some S are not P’ is defined as ‘there is at least one S that is not P’, which would be fulfilled if no S are P, so ‘Some snakes are not elephants’ is considered true. But if you say that the next time you are visiting the zoo everybody will think you are crazier than the chimpanzees.

There is way too much ambiguity here. For goodness sakes, just say what you actually mean! If it is all or none, then say so. Now maybe there could be times when there is some uncertainty about whether the claim should be universal or not, but if so, one could just say: ‘It seems as though all S would be P’ or ‘It is probably the case that no S are P’. Nothing could be deduced from such propositions until more information was discovered empirically, but at least they would be clearly communicated.

A better definition of ‘some’ is ‘not all and not none’ which is how most people use it anyway. Based upon this definition, in order for ‘Some S are P’ to be true, there has to be at least one S that is a P, and at least one S that is not. In other words, part of the subject class has the predicate, and the other part does not.

The Triangle of Opposition

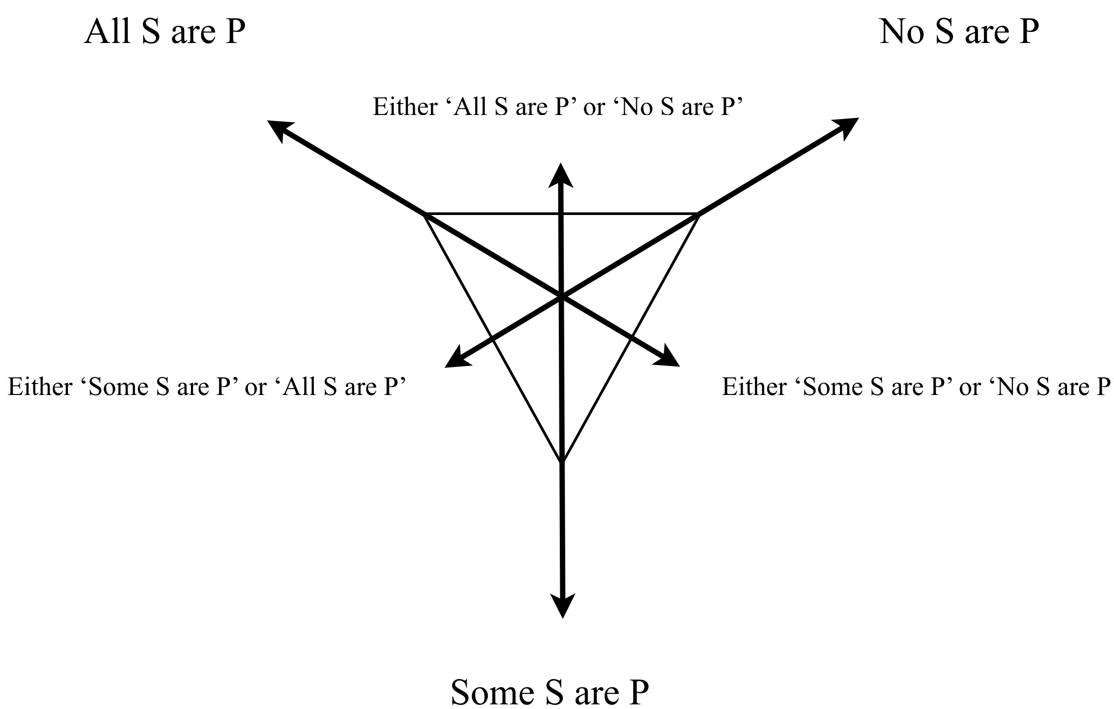
With this definition of ‘some’ the truth value of the two particular statements will always be the same so we could just combine them into one compound statement (although one could also be left implied rather than stated outright, if desired) and create a Triangle of Opposition:



One of these three statements must always be true. (Although there could be times when there is not enough information about the categories to know which.) Each statement is in opposition to the others, meaning that they all have the same subject and predicate but differ in quantity or quality, so if we know which of them is true we would automatically know that the other two are false. However, if all that we are told is that one of them is false then we would know that one of the other two must be true but we could not deduce which one from the given information. If the terms represented categories that we have some familiarity with we could probably tell that way, but it would not be through logical inference.

There are some differences between the Triangle of Opposition and the Traditional Square of Opposition. S*ubalternation* is not valid for the Triangle because with the particular statements merged together into a single proposition a universal could not imply its subaltern without also simultaneously implying its own contradictory (Aristotelian interpretation), which of course is absurd. *Superalternation* is not valid either because if the particular statement is false we would know that one of the universals is true, but we would not know which one.

The three statements are not *contradictories*. The contradictory relation is this:



Each contradictory statement should be understood as an 'exclusive or', meaning that part of it (one of the disjuncts) is true and the other part is false. This means that if 'All S are P' is false it must be the case that either 'Some S are P' is true, or 'No S are P' is true, but not both of them. We would not know which it was without more information. On the other hand, if 'All S are P' is true, then both sides of the disjunction are false for the contradictory, which means that the statement as a whole is false.

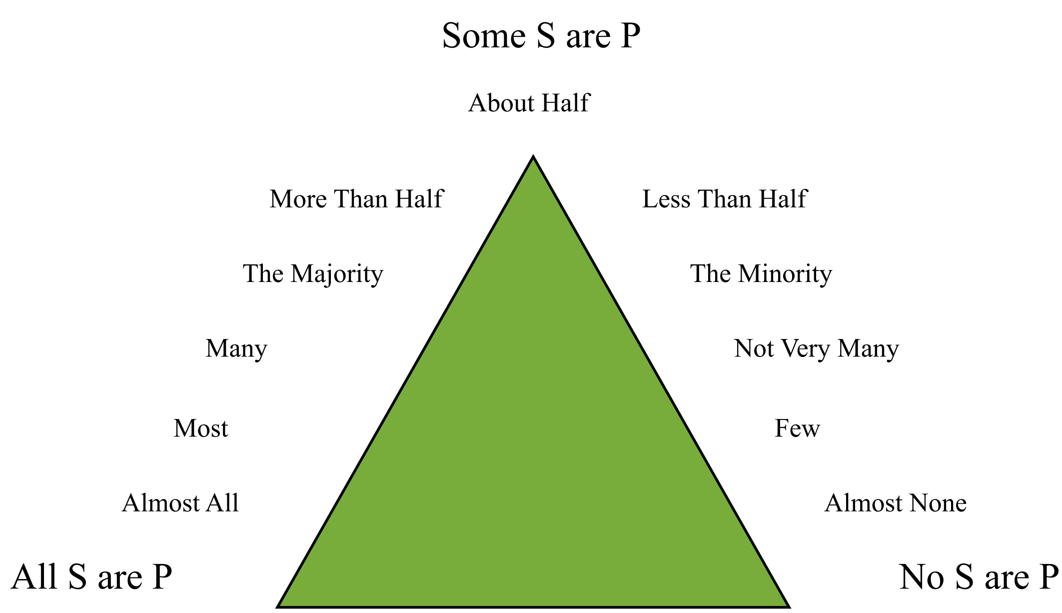
Since an 'exclusive or' is true when one and only one disjunct is true, two of the 'or' statements would always be true. For example, if 'No S are P' is true then 'Either "Some S are P" or "No S are P"' and 'Either "All S are P" or "No S are P"' are also true; the only 'or' statement that is false is the contradictory of ‘No S are P’. However, if all that we were told is that one of the ‘or’ statements is true (which is equivalent to being told that its contradictory is false) we would not be able to tell which other ‘or’ statement is true merely through deduction. Thus, in some cases you could deduce the other five truth values from the one that you were given, but in others you would only be able to deduce the truth value of the contradictory.

The ‘Some’ Spectrum

Consider quantifiers such as ‘few’, ‘many’, ‘most’, ‘almost all’, ‘almost none’, ‘the majority’, ‘the minority’, ‘more than half’, etc. None of these could mean ‘all’, for if it is ‘almost all’ (or ‘almost none’) then clearly it is not ‘all’.

In the past it has been problematic to translate statements that use these quantifiers into categorical form because they are not really equivalent to one of the four standard categorical propositions when using the prior definition of ‘some’. One would have to translate statements like ‘Many S are P’ or ‘A few S are P’ not as an I or an O statement, but as implying both at the same time. Translating them this way is correct, but it does not result in a standard form categorical syllogism, so one could then only check for validity using extended techniques, and actually I do not believe that most of the attempts to do it with extended techniques are even done correctly.

All of these quantifiers are equivalent to my definition of ‘some’, they are just more specific versions of it. Each of them could be translated into categorical form as ‘not all and not none’ or ‘some are, and some are not’. There is a whole spectrum of terms that could stand for ‘some’:



‘All’ is equivalent to 1, ‘None’ is equivalent to 0, and ‘Some’ is equivalent to a fraction or decimal point that is greater than zero but less than one. (0 < Some < 1.) This is also the percentage of S that is P. ‘Some’ is a generic term with an indefinite value, so it could stand for any of these more specific quantifiers, and others as well.

Categorical Syllogisms

I was interested to see how defining ‘some’ as ‘not all and not none’ would affect categorical syllogisms. One way that I tested it was to modify Venn diagrams by simply placing two Xs rather than one in the appropriate locations for particular premises. That works to an extent, but it is not ideal. For one thing, Venn diagrams are based upon Venn’s interpretation of existential import, which I do not share. They also do not show one of the forms which I consider to be valid as valid. So I have developed an alternative way of diagramming.

First things first, though: A, E, I, O are letter names that correspond to the first four vowels of the Roman alphabet and came to be associated with the categorical propositions during the Middle Ages. While we are at it, we may as well update the letters that are used to represent the statements:

A: All S are P
B: Some S are P
C: No S are P

Since for this interpretation there are three kinds of categorical proposition, and three categorical propositions are in a syllogism, there are 27 different moods. There are still four figures (which is determined by the location of the two occurrences of the middle term in the premises), which means that there are 108 forms in total.

I have checked them all and found the following to be valid:

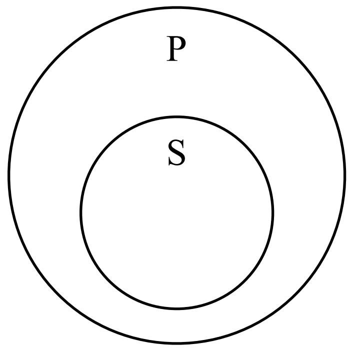
Fig 1	Fig 2	Fig 3	Fig 4
AAA		BAB	AAB
	ACC		ACC
CAC	CAC		

This list may seem sparse, but remember that this is out of 108 forms rather than 256. This interpretation is more restrictive than Aristotle’s, in which 24 forms are valid, which is roughly 9.4%. But it is actually a bit less restrictive than the ‘Boolean interpretation’ in which 15 forms are valid out of 256, which is roughly 5.9%. For me, 7 forms are valid out of 108, which is approximately 6.5%.

Here is how each type of categorical proposition is to be diagrammed and the reasoning for it:

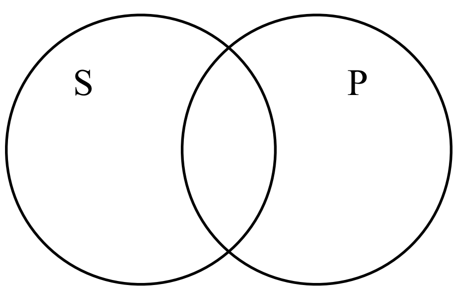
All S are P

Assuming that S and P are not synonymous terms such that all S are P and all P are S (in which case they would have exactly the same members and would just be the same class called by a different name), if all S are P then S has to be a subclass of P. There cannot be any members of S that are outside of P, so the S circle must be entirely contained within the P circle.

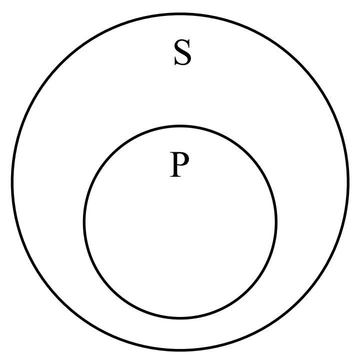


Some S are P

There are two possibilities. We must consider both of them when testing for validity. For the first, they are overlapping classes; for the second, P is a subclass of S, as in: ‘Some animals are dogs’.



or

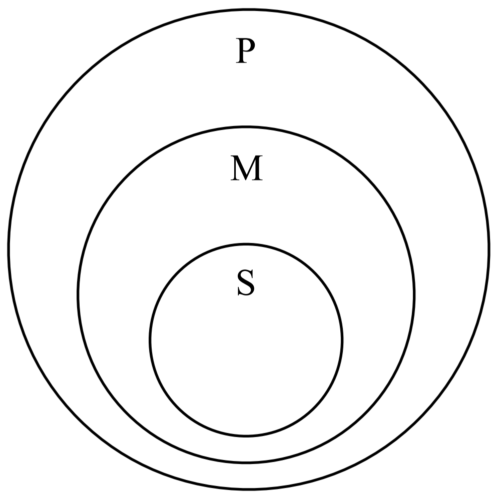


No S are P

In this case the two classes are entirely separate, so we simply have two circles that do not overlap.

The first valid form on the list is AAA-1, commonly known as 'Barbara'.

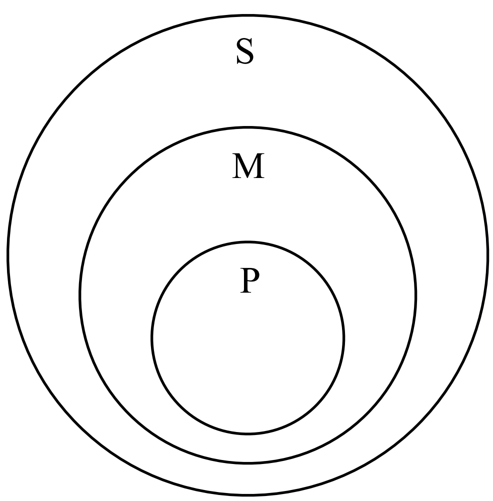
All M are P
<u>All S are M</u>
All S are P

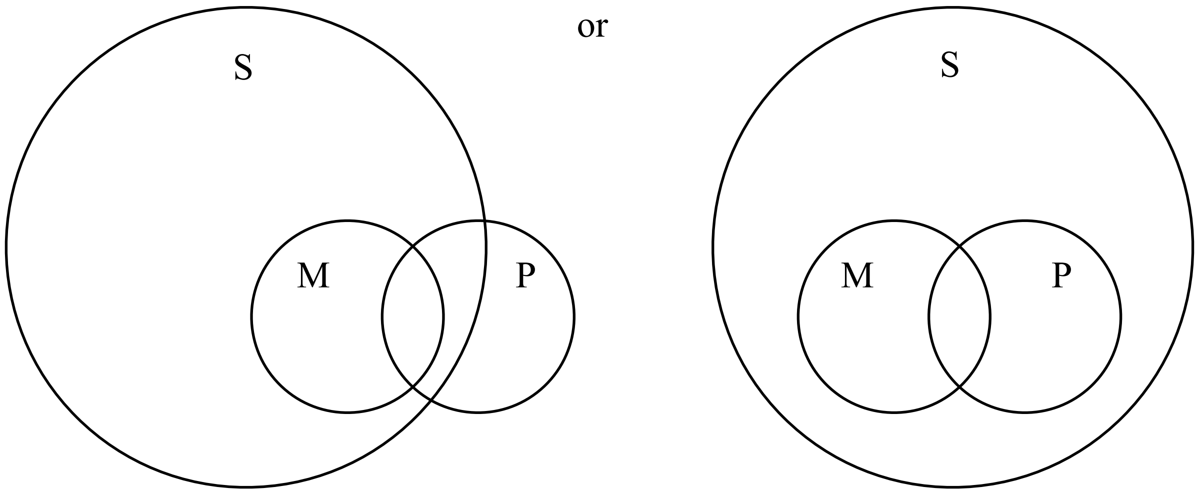


If all S are M, and all M are P, then it has to be the case that all S are P.

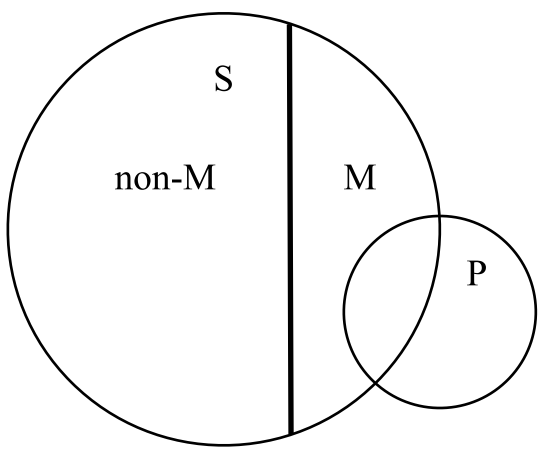
The next valid form I wish to consider is BAB-3. There are at least three possible diagrams for the premises:

Some M are P
<u>All M are S</u>
Some S are P





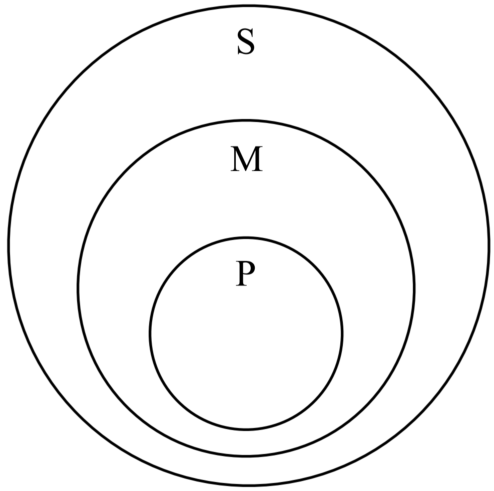
Even though there are multiple diagrams that are possible with these premises, in all of them it is true that ‘Some S are P’. Because some M are P, and all of M is entirely inside of S, it must be the case that some S are P. One could also diagram it this way:



Since some but not all M are P, it would have to be the case that some S are P (some of the ones that are M would also be P) and some are not (which would include the S that are not M, and those that are M but not P). In any possible scenario it has to be the case that some but not all S are P, which is why the argument is valid.

The next form is AAB-4:

All P are M
<u>All M are S</u>
Some S are P



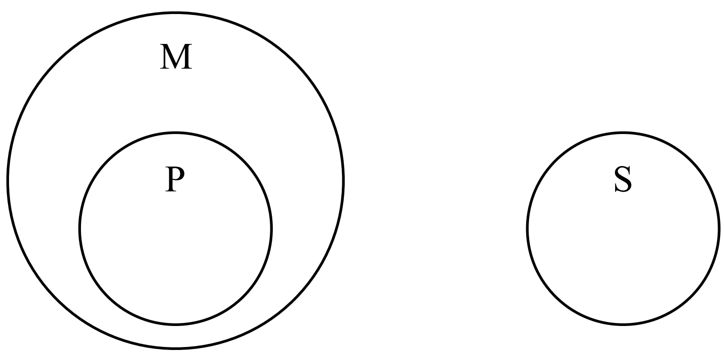
One will notice that this is the same diagram as one of the diagrams for BAB-3. It is the only one possible in this case. If P is a subclass of M, and M is a subclass of S, then P has to be a subclass of S, which is one form of 'Some S are P'. Thus the argument is valid. An example would be:

All dogs are mammals
<u>All mammals are animals</u>
Some animals are dogs

ACC has two valid forms, figure 2 and figure 4. Both have the same diagram:

ACC-2
All P are M
<u>No S are M</u>
No S are P

ACC-4
All P are M
<u>No M are S</u>
No S are P

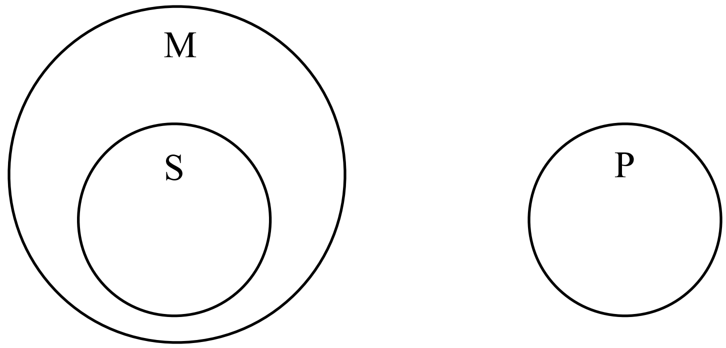


If P is a subclass of M, and no S are M, then there is no possible way that any S could be P.

CAC-1 and CAC-2 are similar. These two forms also have the same diagram:

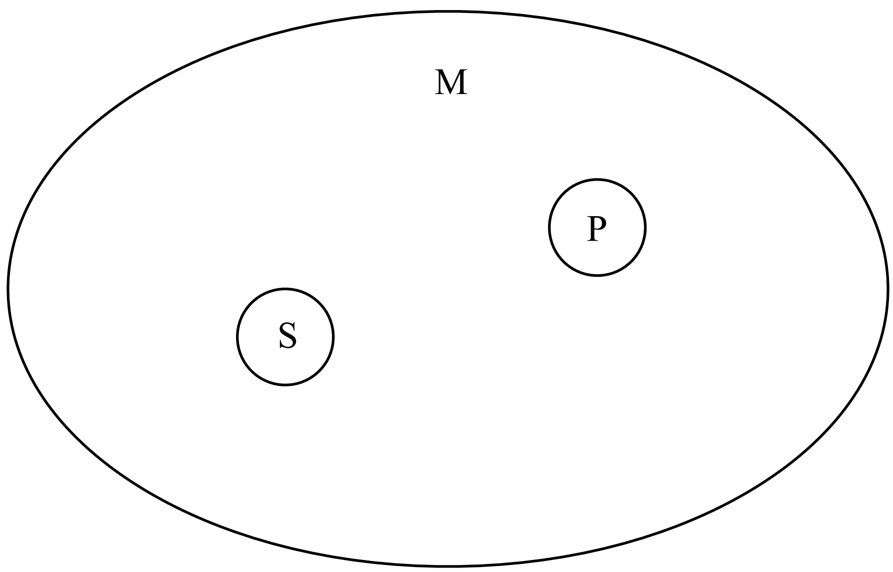
CAC-1
No M are P
<u>All S are M</u>
No S are P

CAC-2
No P are M
<u>All S are M</u>
No S are P



Now let’s look at some invalid forms. For example, the Fallacy of the Undistributed Middle:

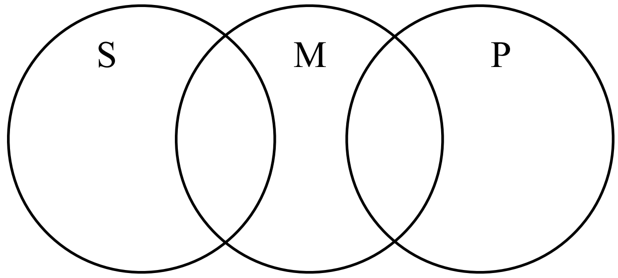
All P are M
<u>All S are M</u>
All S are P



The problem with the argument is that although we have been told that both S and P are subclasses of M we have no idea simply from this what their relation is to each other. Suppose M is a very large class, like ‘mammals’ and S stands for ‘dogs’ and P stands for ‘cats’. Well, it is true that all cats are mammals, and true that all dogs are mammals, but that does not mean that all dogs are cats. However there could be other cases where they do overlap each other, or one could be a subclass of the other (say if one class is ‘bulldogs’ and the other is ‘dogs’) so we cannot say anything with certainty about S and P merely from these premises.

None of the forms having two particular premises are valid. For example, BBA-1 can be shown to be invalid with the following diagram:

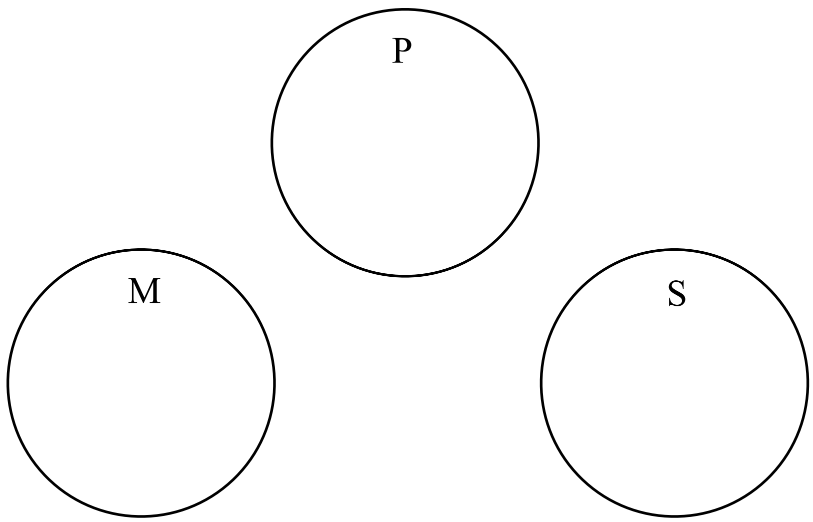
Some M are P
<u>Some S are M</u>
All S are P



There are many other possible diagrams that would be consistent with these premises, but this one is sufficient to show that the form is invalid. In fact, this same diagram could be used to show that all four figures of BBA and all four of BBB are invalid.

Arguments with two negative premises are invalid. For example CCA-1:

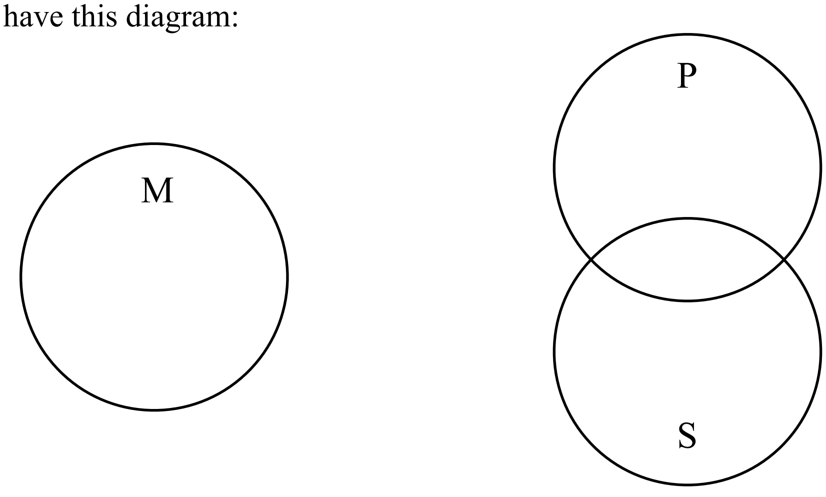
No M are P
<u>No S are M</u>
All S are P



This same diagram can be used to eliminate all four figures of CCA and all four figures of CCB.

For CCC-1 we could have this diagram:

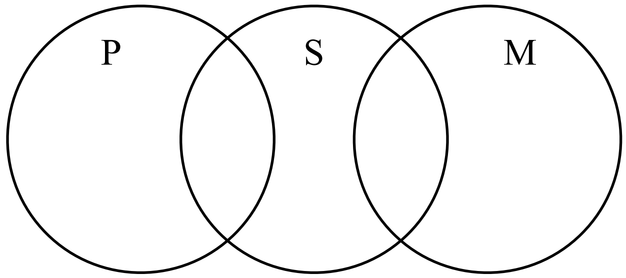
No M are P
<u>No S are M</u>
No S are P



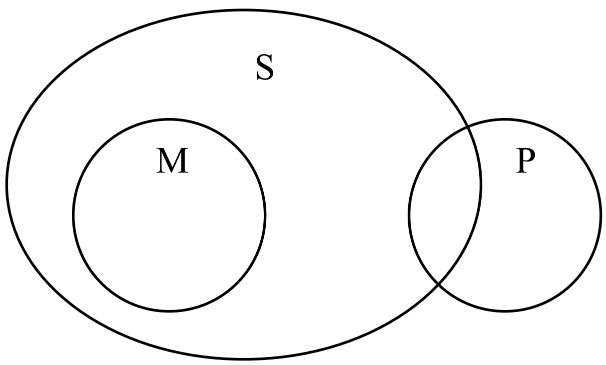
There are other possible diagrams (such as having S or P be a subclass of the other), but this arrangement eliminates all four figures of CCC.

There are at least two possible diagrams that eliminate CBC-1:

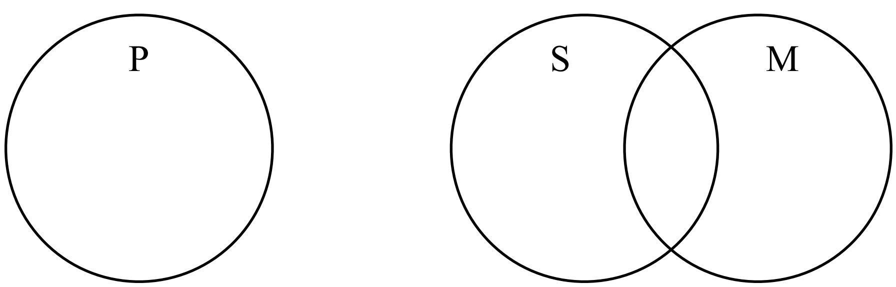
No M are P
<u>Some S are M</u>
No S are P



or



Both diagrams are consistent with the premises, and in both 'Some S are P' is the conclusion that would follow. Thus, CBC-1 is invalid. CBA-1 has identical premises, so the same diagrams show that form to be invalid as well. But the conclusion does not have to be 'Some S are P'. This diagram is also consistent with the premises:



This shows that CBB-1 is also invalid.