The Conditional

By David Johnson

'If . . . then' statements, otherwise known as conditionals, have been giving philosophers fits since ancient times. There has been a great deal of debate concerning the proper interpretation of them. However, I have found no interpretation that I am satisfied with, and will therefore provide my own.

To begin, we ought to first be clear about what a 'condition' is. There are a number of senses in which the term is used in everyday language, but the relevant one for our purposes is: a state of affairs that must exist or be brought about before something else is possible or permitted. One may come across statements such as: 'I'll accept on one condition', 'All personnel must comply with this new policy as a condition of employment', or 'For a member to borrow money, three conditions must be met', and so on.

There are two types of conditions. If P is a *necessary condition* for Q, then Q cannot be realized without P. However, P being realized does not guarantee Q. Water is necessary to sustain human life, but it is not the only thing required. One could not guarantee that there is human life merely from the fact that there is water. A necessary condition is kind of like an essential ingredient in a recipe, but of course there could be other ingredients that are essential as well. If the realization of P did guarantee Q, then P would be a *sufficient condition* for Q. Being female is a necessary (but not sufficient) condition to be a mother; being a mother is sufficient (but not necessary) to guarantee that it is female. If you are in Nebraska that would be sufficient to guarantee that you are in the United States, whereas it is necessary to be in the United States in order to be in the state of Nebraska.[1]

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[1] An interesting debate related to this topic is how to interpret Plato and Aristotle concerning the connection of virtue to eudaimonia (which means happiness and/or well-being, or flourishing). It is clear that both of them believed that virtue was a condition for eudaimonia, the question is what kind of condition. My personal take on the subject is that for Aristotle virtue is a necessary condition. In other words, you cannot have eudaimonia without being virtuous. In most cases, Aristotle says, if you are virtuous you will have eudaimonia, and it would be difficult to take it away from a good person, but he does leave open the possibility that in really extreme circumstances, such as those endured by Priam in the *Iliad*, a person could be virtuous and still have their life turn out tragically because of circumstances that are out of their control. Thus, having virtue is not sufficient to guarantee eudaimonia, it only makes it possible. For Plato, virtue is both a necessary and a sufficient condition. He thinks that if you have virtue you are guaranteed to have eudaimonia (virtue is sufficient), and you will have eudaimonia only if you have virtue (virtue is necessary). In other words, you will have eudaimonia if and only if you are virtuous. I interpret him this way because in the dialogues Socrates makes such claims as: a good man cannot be harmed, it is better to suffer injustice than to commit injustice, etc. He also seems to indicate that he thinks it is possible for someone to have eudaimonia even if he has been falsely accused of a crime and is being tortured as long as he is still virtuous. This all leads me to think that he believes that being virtuous is sufficient for eudaimonia. But he also thinks that virtue is necessary as well. In *The Republic* he says that the unjust tyrant is the most miserable of all, even though he has all of the things that most people believe would make them happy. The only thing that the unjust tyrant lacks is justice / virtue, which would indicate that without it one cannot have eudaimonia.

In addition to stating conditions as they are given in the examples above, people often use conditional statements. In its most standard form this is a claim that one part of the statement, the antecedent (which follows ‘if’) is a sufficient condition for the consequent (which follows ‘then’). However, a necessary condition is often stated in the consequent. For example, conditional statements can be used to indicate what a person must do to obtain some desirable outcome or reward. Meeting the condition is the means to obtaining that end. In these statements, known as hypothetical imperatives, the end which is desired is usually stated in the antecedent, and then the consequent identifies a condition that must be met in order to obtain it, such as: ‘If you want to have long-term success in business, you have to be honest.’ Here it is claimed that being honest is a necessary condition for having long-term success in business - too bad more business people do not believe it.[2]

Conditional reasoning is sometimes confused with causal reasoning. There are some similarities, but conditional reasoning is broader and has a higher level of abstraction. X can be a condition for Y without necessarily being the cause of Y. ‘If it is a three-sided figure then it has three sides,’ for example.

David Hume thought that all of our knowledge of causation comes from the ‘constant conjunction’ of events; we observe that they always, or nearly always occur together, and from this conclude that one causes the other. His overall point is that our knowledge of causation is not *a priori*, it comes from experience. That part is true, but describing causation as a conjunction is not entirely accurate. He does say ‘constant conjunction’, so it would have to be more than just a random pairing that only happens occasionally, but even this would not be enough to say that one physically causes the other. What if both A and B happened as a result of C, which is the true cause of both, but C is rarely, if ever observed?

Another problem is that in a conjunctive statement, the conjuncts can come in either order and the truth value remains the same: ‘A • B’ or ‘B • A’ are equivalent (the dot symbol stands for ‘and’). That is certainly not the case with causation. Consider the simple example of turning on the lights. In my experience, whenever I flip the light switch to the ‘on’ position, the light comes on. (Unless of course there is some problem with it, such as a power outage, or a light bulb is burned out, etc.) So it is natural to believe that flipping the light switch is what is causing the lights to come on. If it was merely constant conjunction sometimes the lights could come on first, and then the switch would be flipped, or they could happen at exactly the same time. But if A is truly the cause of B, then A must always precede B. This is because B, the effect, happens as a result of A. There could theoretically be a cause that is simultaneous with its effect(s), and some philosophers have speculated that indeed there is, but all of the causes that we are familiar with in our everyday experience always precede their effect in time.

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[2] Correct symbolization of this claim would be $\sim H \rightarrow \sim S$; see the section on biconditionals for more information.

Constant or frequent conjunction actually corresponds more to correlation than causation. Hume did not think that we could tell the difference between them because we would experience both the same way, but that is not entirely accurate. The fact is that we can and do make that distinction all the time. The main differences are that cause always precedes effect, whereas when two events are merely conjoined they can occur in any order, and two events can be correlated without necessarily having anything more than a random connection to each other. In fact, a common characteristic of flawed causal reasoning is that it identifies events that are correlated, but neither one is actually the cause of the other. A preceding B or being associated with B does not necessarily guarantee that A is the cause of B. To mention a classic counterexample, just because the rooster always seems to crow right before, or as the sun comes up, does not mean that the rooster crowing caused the sun to come up. A preceding B is necessary, but not sufficient, for A to be the cause of B.

Causation corresponds more to a conditional: just as cause must always precede effect, and brings about the effect, so also the condition is logically prior; the resulting state of affairs, it is claimed, is dependent in some way upon that condition being met. However, the truth table for what has come to be known as a 'material conditional' does not recognize this dependence, and that is one reason (among others) why it is defective.

According to the truth table, a conditional statement is logically equivalent to ~(A • ~B). Thus, 'If it is raining then the streets are wet' would be equivalent to 'It is not the case that it is raining and the streets are not wet'. But ~(A • ~B) does not express a conditional relationship between A and B, it only expresses the negation of the conjunction of A and ~B. The claim that A and ~B are not conjoined is not the same as the claim that A is a necessary or a sufficient condition for B. In a conjunction, the presence of one conjunct does not guarantee the presence of the other, nor is the presence of one a requirement for the other, it is just two simple propositions conjoined in that particular case, and that is all. But of course, that is not the case for a legitimate conditional. If A is a sufficient condition for B then if you have A you *must* have B, and if it is a necessary condition then if you do not have A you *cannot* have B. There is necessity in those claims, it is not merely a random pairing of two simple statements. Admittedly though, I have tried to think of a counterexample in which A⟶B, or 'If A then B', is obviously true while ~(A • ~B) is clearly false, and I have not been able to come up with a good one; this indicates that when the former is true the latter is as well. However there are many instances in which ~(A • ~B) is true, but A is not really a condition for B. To see examples of this, one need look no further than the first line of the table:

Antecedent	Truth value of whole statement	Consequent
1. T	**(T)**	T
2. T	**(F)**	F
3. F	**(T)**	T
4. F	**(T)**	F

The first line says that whenever the antecedent and the consequent are both true the conditional as a whole is true. But take the statement: ‘If Immanuel Kant was German then cows are herbivores’; yes, the antecedent and the consequent both happen to be true, but they are each true independently of the other. It may indeed be correct that, ‘It is not the case that Immanuel Kant was German and cows are not herbivores’, but that certainly does not mean that Immanuel Kant being German is a *condition* for cows to be herbivores.

The methodology of truth-functional logic, which this table is based upon, is that the truth value of the whole statement is solely determined by the truth value of its component parts. However, in the case of conditionals it is actually somewhat the opposite. If the antecedent is true, and it is a sufficient condition for the consequent, then that is the underlying reason why the consequent is true.

A merely random pairing of simple propositions that are true, but true independently of each other, would, if conjoined, result in a true conjunctive statement; but obviously, if the consequent is true independently of the antecedent, then the antecedent must not really be a condition for the consequent. We would not consider a causal inference to be correct if there was no actual causal relation between the purported ‘cause’ and ‘effect’, and that would be the case even if both were true (such as the rooster really did happen to crow just as the sun was coming up).

Reasoning of this sort is similar to the informal fallacy of *drawing the wrong conclusion* in an argument. In this fallacy, the premises, which are offered as evidence in support of the conclusion, are in fact entirely irrelevant to the conclusion; they either imply some other conclusion, or perhaps nothing at all. Such an argument would not be any good even if those premises and the conclusion were all true.

However, it should be noted that according to the truth-functional interpretation, arguments are considered valid as long as there is not a line on the truth table in which there are all true premises and a false conclusion, even if the premises are irrelevant to the conclusion. Thus the following argument would be valid:

All men are mortal
<u>Washington D.C. is the capital city of the United States</u>
Therefore, it is either raining or it is not raining

The conclusion is a tautology (meaning that it must always be true) so it would be impossible for the argument to ever be considered invalid according to the truth-functional interpretation. It would not even matter whether the premises are true, but in this case they are, so not only would the argument be valid, it would also have to be considered sound. But of course it makes no sense to say that it is valid. It is merely a random grouping of propositions with no relation between them. The whole point of stating a premise is to provide some type of support for the conclusion. Here the conclusion is true, but entirely independently of the premises.

Another odd example that the truth-functional interpretation would consider valid is when there is a self-contradictory premise, or two premises that contradict each other, such as the following:

It is day
It is not day
Therefore, Paris is the capital of France

This argument is considered valid because there is not any line of the truth table in which both premises are true: since they directly contradict each other, they must always have opposite truth values. In this case the conclusion is true, but it does not matter, the argument would be valid no matter what conclusion is used. Another way that this could happen is when an argument contains a self-contradictory premise; such a premise could never be true on any line of the table. As in:

It is raining and it is not raining
Thus, either it is raining or it is not

This would be the king of all arguments for truth-functional logic, apparently, because the premise is self-contradictory and the conclusion is a tautology. But, of course, in no sense could the conclusion be said to *follow* from the premise. If the concept of an argument is to offer premises as reasons to believe the conclusion, this is an awful argument.

These are like Gettier examples of validity - they should not even be considered valid at all. A self-contradictory statement could not ever be a legitimate premise; if it is never true itself it cannot be support for any other claim; and, if the premises contradict each other, then together they could not be considered good support for any conclusion. A tautology is a self-evident claim that needs no additional support; any argument one might make for it would be superfluous, and in fact no premise would support it more than any other; analyzing it as the conclusion of an argument is like making a category-mistake.

In a legitimate valid argument, the premises logically imply the conclusion. In a similar way the antecedent implies the consequent in a conditional, but only if it is a genuine condition. 'If the city of Paris is in France, and the chemical symbol for the element Silver is Ag, then all snakes are reptiles' is not a legitimate conditional. It is not a good *inference* from the antecedent to the consequent because the supposed 'conditions' stated in the antecedent are not genuine conditions for all snakes to be reptiles.

The second line of the truth table says that when the antecedent is true, and the consequent is false, the entire statement is false. This is for the most part correct, but there are at least two problems: 1) the truth or falsity of the conditional statement is not determined by the truth values of the antecedent and the consequent, and 2) it assumes that the antecedent is always purported to be a sufficient condition. The underlying reason for why this line of the truth table holds up better than the others is that there is no way that the antecedent could be considered a legitimate sufficient condition for the consequent if the antecedent is true (meaning that the stated condition has been met) and the consequent is false. It usually works because most conditionals are structured this way.

However, if the antecedent is a necessary (but not sufficient) condition[3], as in: 'If you live in the United States then you live in the state of California' there is a certain probability that the claim would be correct. The consequent would be true for some people who have met the condition (namely Californians), but false for those who live in any other state. The truth table does not account for conditionals that have a probability between 1 and 0.

Line 3 is where it really starts to get weird. On the third line the antecedent is false, the consequent is true, and the statement as a whole is supposedly true. As with line 1, entirely random pairings of propositions would be permitted. 'If pigs could fly then 7 is a prime number' would be an example. Is pigs being able to fly a state of affairs that must be realized in order for 7 to be a prime number? Clearly not, as that state of affairs has not been realized, and yet 7 is a prime number. Pigs being able to fly is neither necessary nor sufficient for 7 to be a prime number, so why, if it is not really a condition for the consequent, would a conditional statement claiming that it is be considered true?

If the simple proposition A is true, let's say it represents 'Aristotle was a philosopher', then $A \supset {\sim}A$ is considered false, being an example of line 2 of the table. This is perfectly reasonable because it is saying 'If Aristotle was a philosopher then it is not the case that Aristotle was a philosopher' which of course makes no sense. However, if the order were changed to ${\sim}A \supset A$, or 'If it is not the case that Aristotle was a philosopher then it is the case that Aristotle was a philosopher' then it would be an example of line 3, and is considered true. But that claim makes no sense either. I realize that the condition has not been met, but the two simple statements are contradictories. If they always have opposite truth values (and they must) then there is no way that one could ever be a condition for the other. Something that is contradictory to the consequent cannot be a condition for the consequent because if the condition is met then the consequent cannot possibly be true; we know that analytically, whether the condition has been met or not.

In the case of the fourth line, any two false propositions can be paired together, even if they have no relation at all to one another, and the conditional statement is supposedly true. 'If the symbol for the chemical element silver is Br then David Hume was an extraterrestrial' would be an example. All I can say is really? We are really going to say that is true? If Hume was an alien then perhaps he assisted with the construction of the pyramids (as some theories assert). In fact, we could make it so: 'If David Hume was an extraterrestrial then he created the Egyptian

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[3] Because of how this claim is stated, I suppose one could consider 'living in the United States' to be a purported sufficient condition for 'living in California', according to the claim, even though it is not really sufficient. Either way, the claim would have a certain probability. It may be a relatively low probability, but still a probability. It would not be correct to say that it is false any more than to say that it is true.

pyramids in one day using only his extraordinary mental powers'. These can be kind of fun, I admit, but it is hard to understand how anyone could really take them seriously. The truth table does, though; both claims would be true according to that - which makes it hard to take the truth table seriously.

We could summarize the last two lines by saying that according to the truth table, whenever the antecedent is false the conditional as a whole is always considered true no matter what. Even claims like: 'If Socrates was alive today then he would be a goddess' and 'If there were leprechauns then they would be both short and not short' and 'If there were actual unicorns then the term 'unicorn' would be defined as a three-horned creature', etc. The truth table indicates that all of these are true (!), being examples of the fourth line, but of course we know that for all of them the consequent would never be true (as it is a self-contradictory proposition itself), even if the antecedent was.

It gets even more bizarre. An implication of the truth table is that if the antecedent is self-contradictory (or if the consequent is a tautology) then the conditional would not just be true, but necessarily true: 'If it is both raining and not raining then insects are reptiles' would be some sort of strange tautological statement. But of course it would not be limited only to that; any consequent would apparently follow from 'If it is both raining and not raining . . .' no matter how strange or silly the claim ends up being. It was bad enough that 'If insects were mammals then A and not A' was considered true, but now 'If A and not A then insects are mammals' is apparently a necessary truth. I, for one, would like to know why 'A and not A' would be a condition for insects to be mammals, (or vice versa) even theoretically.

Suppose that we had two claims which said the following: 'If it is 10 a.m. then y' and 'If it is 10 a.m. then not y'. At precisely 10 a.m. each day, at least one of these claims would have to be false, and the truth table would show that. But what about at 10:01, or 5 p.m., or midnight? Can't we say for sure that they cannot both be true even if it was some other time of day? It seems to me that would be *a priori*. They could both be false (say if it being 10 a.m. was not a legitimate condition for either), but there is no way that the same condition could really imply two directly contradictory propositions. Yet the truth table indicates that it is impossible for us to know that except when the condition is met. Both claims would have to be considered true at any time other than 10 a.m. because the antecedent would be false at any other time.

The truth table also does not adequately account for conditional statements that are based upon physical causation. For example, the claim 'If I eat a cupcake then the earth will explode' is quite obviously not true. But suppose that I was superstitious, and believed it, and because of that never at any time ate a cupcake. That means that the antecedent would always be false and the conditional as whole would be considered true even though it is completely ridiculous. My eating a cupcake is not a necessary or a sufficient condition for the earth exploding, and we all know that whether or not the 'condition' has been met. In fact, this would be an example of the fallacy of *false cause*. A false antecedent does not guarantee a true conditional. One should not just assume that a claim is true simply because it has not yet been proven false. Not all unconfirmed claims are equal. Some hypotheses, even if untested, are much more plausible than others. The probability of this one would be very close to zero, but others could be quite likely, or even certain if the antecedent condition were met.

Truth tables are part of propositional logic. The idea of a truth table is that one can analyze the individual parts of a statement and then derive from their truth value the truth value of the whole compound statement. This works for conjunctions and disjunctions (‘and’ and ‘or’ statements respectively) because they are compound propositions. However it does not work for conditional statements because a conditional is *not* a proposition. A proposition, one may recall, is a claim that can only be true or false: the law of excluded middle states that for any proposition, either it is true, or its negation is true. It cannot even have some degree of probability, it can only be true or false. Consider a conditional such as: ‘If I toss a fair coin then it will come up heads.’ Is that true or false? Both answers seem equally correct, or perhaps more accurately, equally incorrect. If this is a proposition, I am not sure what its truth value would be.

Despite the absurdities that it produces, it is somewhat understandable why the truth table is set up the way that it is. If true or false were the only options available, I suppose one would probably have to say that the whole statement is true whenever both antecedent and consequent are true, even if they are only paired through random chance. If it is not considered true when both component parts are true, I am not sure when it would be. There would be no way to distinguish between instances in which it just happens to be the case that both simple statements are true, and instances in which they are both true because the antecedent is a legitimate condition for the consequent. Similarly, one would probably have to say that a conditional statement cannot truly be considered false unless the antecedent is true, and the consequent is false, so whenever the antecedent is false, or the consequent is true, the entire claim would have to be considered true rather than false, if those were the only options. I could not come up with a better truth table, based upon the assumption that conditional statements are propositions, but the inadequacies of the table show that an entirely different kind of analysis is needed.

Conditional statements are really much more like arguments than propositions. They are not arguments, but are of the same genus, both being a type of inference.[4] In arguments, we make a distinction between validity and soundness, strength and cogency; it is recognized that there are two separate considerations involved, one being whether the conclusion follows from the premises (or how strongly it does in the case of inductive reasoning), and the other being whether the premises are true. A similar distinction is needed for conditional reasoning. Like an argument, one could not really say that a conditional is ‘true’ or ‘false’ because the claim is more complex than that; there are really two considerations involved, the primary one being whether the antecedent really is a condition for the consequent, and/or whether the consequent ‘follows’ from the antecedent, as is claimed, and the second is whether that condition has been met; if it

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[4] The greatest similarity is to causal reasoning. Just as ‘causal reasoning’ differs from causation itself, the former being an inference based upon the latter, so also ‘conditional reasoning’ differs from a condition, but is based upon it.

has, the claim purports to accurately reflect an actual state of affairs; if not, then it is merely a hypothetical conjecture of what would follow if the condition was met. Both of these considerations are necessary in order to evaluate the claim correctly.

I find it interesting that in math what follows 'if' is not referred to as the antecedent but as the 'hypothesis' and what follows 'then' is not called the consequent but the 'conclusion'. That makes it sound an awful lot like an inference. And yet, since what they are referring to is the material conditional, with the same truth table as the one given above, it will then be carefully explained that there is not necessarily any causal or logical connection between the 'hypothesis' and the 'conclusion', despite what those terms imply. Well, there ought to be, and indeed there is in a legitimate conditional. Usually after giving an example such as 'If it is a unicorn then it would be a five-horned creature', which is considered true (!) according to the truth table, there is some sort of admission by the author such as '. . . the logical meaning of the material conditional is not the same as its intuitive meaning,' or, from a writer in logic, 'The statement is true in logic, but by the standards of ordinary language it is not.' These rather uncomfortable acknowledgements are reminiscent of the 'double-truth' theory of the Middle Ages - and are no more convincing.[5]

Conditional reasoning, causal reasoning, and arguments are all types of inferences. Of the three, conditional reasoning is the most broad; in fact, it can easily overlap with the other two. Causation and arguments can both be described in terms of conditional statements. If A is the cause of B, then A's occurrence would be sufficient to guarantee B's occurrence; and, of course, if it is not the case that the effect (B) is observed, then it must also be the case that the cause did not occur. Similarly, the premises of a valid argument could together be expressed as a sufficient condition, that if met, or if true, guarantees the consequent, which would be equivalent to the conclusion. It would also be the case in a truly valid argument that if the conclusion is false, one of the premises would also have to be false, which is similar to a necessary condition. Many conditional statements could also easily be expressed as arguments, with antecedent as premise, and consequent as conclusion.

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[5] While on the subject of mathematics, sometimes in a math proof it is helpful to be able to show that a conditional statement is, or would be false (or I would say not implied). If the conditional statement is 'If A then B' then the method that is employed to do this is to find an instance, either directly or through a series of valid steps, a case of A • ~B. This is the perfect method to use to prove that a material conditional is false, because, as one will notice, it is the exact opposite of ~(A • ~B). It is basically finding an instance of the second line of the truth table. It would also be effective in proving that A is not really a sufficient condition for B. However, it should be remembered that this would not disprove that A is a necessary condition, or that the antecedent implies the consequent with some level of probability less than 100% but greater than 0. Suppose that we had a conditional equivalent to 'If x is a male then x is a brother'. Finding a instance in which something is male and not a brother, M • ~B, would not disprove that M is a necessary condition for B. To disprove a necessary condition, one would need to find an instance of ~M • B (one does not exist in this case because it is a necessary condition). Thus, for 'If A then B', an instance of A • ~B would disprove that A is sufficient (at least with 100% probability), and an instance of ~A • B would disprove that A is necessary. Both would be needed to show that A is neither necessary nor sufficient for B, if that is required. Finding an example of either one would disprove a biconditional, such as 'B if and only if A', if that is what is sought.

However, in an argument, the assumption is that the premise(s) is/are true. In a conditional inference it is not necessarily the case that the antecedent is true. In fact, one of the most common uses of conditional inferences is to present counterfactual scenarios that allow one to explore the implications of what would happen if some other hypothetical state of affairs existed rather than the actual one. The claim is that the consequent state of affairs would follow from the counterfactual scenario that is proposed in the antecedent. Even if the antecedent is not specifically counterfactual, it might merely be possible, such as a prediction about the future.

Whether the antecedent condition has been met is a similar consideration to whether the premises are true in an argument. An argument can still be valid, even with one or more false premises. For example:

New York City is the capital city of the United States
<u>I am in New York City</u>
Therefore, I am in the capital city of the United States

The argument is valid because the conclusion follows from the premises, but of course the first premise is false, so even if the second one were true the argument would be unsound. In a conditional inference, if the condition has not been met it is like an argument in which at least one of the premises is false.

In some cases we can tell, or at least give a good guess as to whether the antecedent would be a condition for the consequent even if the condition has not been met. But this is an unconfirmed assertion, much like an untested scientific hypothesis. It is a trap that has been set, but not triggered, and the only thing that can trigger it is the condition being met. Until then, it is merely hypothetical conjecture. If the condition has been met, the inference is actual, if not, then it is hypothetical.

The other consideration is whether the consequent follows from the antecedent, and if it does, how strongly. If the antecedent is a genuine sufficient condition for the consequent, then the conditional inference is 'entailed', meaning that the antecedent state of affairs entails the consequent. (A definition of entailed is 'to have, impose, or require as a necessary accompaniment or consequence'.) This is equivalent to 'valid' for an argument.

'Implied' means that meeting the condition stated in the antecedent implies the consequent with some level of probability between 0 and 100%, but not including either one. This is somewhat equivalent to 'strong' 'moderate' or 'weak' for inductive arguments. In this case the consequent has a certain probability of being true (or realized) if the condition stated in the antecedent is met. In other words, we can infer the consequent from the antecedent with some degree of probability. To return to a prior example, the best answer for 'If I toss a fair coin then it will come up heads' is to say that the truth of the antecedent moderately implies the truth of the consequent.

There is a continuum of ‘implied’ claims. Entailment could be considered implication of 100%, whereas when the antecedent contradicts the consequent it would not imply the consequent at all. Anything in between would be implied to some degree.

entailed	100% likelihood
very strongly implied	90-99.99%
strongly implied	75-89.99%
moderately implied	25-74.99%
weakly implied	0.01-24.99%
not at all implied	0% likelihood

Suppose that instead of saying ‘If it is raining then the streets are wet’, which would be entailed, we said ‘If the streets are wet then it is raining’. Because the streets being wet is a necessary rather than a sufficient condition for it to be raining, it is possible for the antecedent to be true and the consequent false. Perhaps there is a street cleaner washing the streets, or someone is watering their lawn and the sprinkler is hitting the street, or maybe someone is washing their car and the water is running into the street. But most of the time if the streets are wet it is indeed because it is raining. The best answer would be that the truth of the antecedent very strongly implies the consequent.

It will also be a matter of probability if the conditional inference is based upon a causal relationship. For instance, ‘If you smoke then you will get cancer’. The claim suggests that smoking is a sufficient condition for getting cancer but the antecedent is not actually sufficient to guarantee the consequent in a technical sense because it is possible to find a few outlier examples in which someone smoked three packs a day for 30 years and never got cancer. (It would not help any if smoking was considered a necessary condition because then it would have to always be true that if you do not smoke you will not get cancer, which is obviously not the case.) But most of the time it would be true, there are just a few outliers that are exceptions to the general rule. It certainly would not be correct to say that the conditional is false, because based upon empirical evidence, if you smoke the probability is significantly higher than it would otherwise be that you will get cancer. The correct answer is that the antecedent strongly or very strongly implies the consequent.

For most conditionals that are based upon causal laws it would be possible to find at least one outlier example if one really wanted to take it to the extreme. Even for ‘If it is raining then the streets are wet’ it would be possible (though quite absurd) to cover up the streets specifically in order to prevent them from getting wet when it rains. To some extent, we are assuming normal conditions with an inference like that.

Causal laws do not have 100% probability of holding in the future, so one could consider conditional inferences that are based upon them to be very strongly implied rather than entailed, but it is close enough to 100% that the chance of failure would be statistically insignificant for most purposes. I would consider it entailed, or we might say ‘practically entailed’, if there is less than 1/100th of a percent chance of failure assuming normal conditions. (The same could be said for ‘validity’ in arguments that are based upon empirical evidence and causation.)

To review, the distinctions would be *entailed* or *implied* if the consequent follows from the antecedent, and *hypothetical* or *actual* depending upon whether the antecedent is true. If the condition has been met the claim is actual. If not, or if it is not known whether the condition has been met, it should be considered merely a hypothetical conjecture. One would not normally need to state a condition if it is obvious that it has been met. ‘If there were horses then all of them would be mammals’ is not incorrect, but ‘All horses are mammals’ is more succinct; since it is a well-known fact that there are horses, we may as well just use the latter. But if the claim is intended to be hypothetical it should be stated as a conditional for the sake of greater clarity. When the condition has not been met but the consequent does follow it should be referred to as ‘hypothetically implied’ or ‘hypothetically entailed’.

hypothetically implied	actually implied
hypothetically entailed	actually entailed

I myself have referred to conditionals as ‘hypothetically true’ or ‘hypothetically false’, particularly those which contain a complete categorical proposition as the consequent, such as: ‘If there were unicorns then all of them would be one-horned creatures’, but that is not entirely correct because a conditional is not a proposition. To clarify, the truth value should be understood to be referring to the categorical proposition only. It is equivalent to saying ‘based upon the hypothetical supposition that there were unicorns it would be true that all of them would be one-horned creatures’. The purpose of the antecedent is to identify the supposition that must be made in order to evaluate the categorical claim. This supposition is made the condition, that if met, would make it possible for the categorical proposition to be true or false. ‘All unicorns are one-horned creatures’ has no actual truth value because ‘unicorns’ is not a category with actual members in the real world. Of course even in conditional form the condition has not been met in the actual world, meaning that the claim is like an argument with a false premise (or at least a premise that is not actual) and a conclusion that is false or undefined in actuality but would follow from the premise. It is really a type of counterfactual: the categorical proposition is in actuality undefined because the category does not have actual members, but supposing that it did, this is the hypothetical truth value that the proposition would have.

One can refer to the consequent as true or false in much the same way that one would refer to the conclusion of an argument as true or false, but just as it would be incorrect to refer to the argument itself that way, so also with a conditional; it is an inference, which does not have a truth value. With this kind of claim it would probably be most convenient to refer to the consequent only, in most cases, and say that it is hypothetically true or false. One could even use Aristotle’s Square of Opposition. All the same relationships would hold, it would just need to be noted that the truth values are hypothetical only. Some values would be unknown, such as ‘If there were unicorns then some of them would be black’. The categorical proposition would probably be true under the given condition, but it is not known for sure. One could either say that it is probably true (hypothetically), or just say that it is undetermined. Undetermined differs from undefined in that a claim which is undetermined has a truth value, it is just currently unknown what it would be, whereas one that is undefined has no truth value at all.

For this type of conditional, whether the antecedent implies the consequent really just depends upon whether the categorical proposition would be true, or how often it would be. If it would always be true when the condition is met then the conditional itself would be entailed, if only sometimes true then it would be moderately implied, etc. ‘All unicorns would be white, if there were unicorns’ is a hypothetical inference that is only weakly implied because it is unlikely that the categorical proposition would be true if the condition was met. However, ‘Some unicorns would be white, if there were unicorns’ is strongly implied, and ‘If there were unicorns then all of them would be one-horned creatures’ is hypothetically entailed.

All categorical statements could be expressed conditionally, with the condition being that the category has members, but it is not necessary to do so if it is obvious that the condition has been met. ‘All dogs would be mammals if there were dogs’ could be classified as actually entailed, or we could say that the consequent is true in actuality, but since it is obvious that the category has members one could just use ‘All dogs are mammals’ and say it is true. It should be stated conditionally and given a hypothetical truth value when the condition has not been met, however, in order to avoid confusion.

‘If it is a dog then it is a mammal’ is also entailed, but it is not entirely logically equivalent to ‘All dogs are mammals’ because the former is an inference from one simple proposition, ‘It is a dog’, to another, ‘It is a mammal’ while the latter is a single proposition that is either true or false. But the two claims do logically imply one another, or in other words, the truthfulness of one does imply the truthfulness of the other. The only way that the conditional could be entailed is if in fact all dogs were mammals, and if it is the case that if something is a dog then it is a mammal then it must also be true that all dogs are mammals. But we would not refer to the conditional as ‘true’ or ‘false’ as we would for the categorical statement, we would say that if something is a dog that entails that it is also a mammal, because all dogs are mammals. Conditionals such as this would be claiming that being a member of the subject category is a sufficient condition for being a member of the predicate category. (A conditional equivalent to the universal negative would be claiming that membership in the subject category is sufficient to guarantee that it does not have membership in the predicate category.) If the sufficient condition is sometimes, but not always sufficient to guarantee the consequent, this would mean that the claim has some level of probability and that a particular categorical claim is true. For example, ‘If it is a horse then it is fast’ is moderately implied because some horses are fast and some are not. This should be distinguished from ‘If there are horses then some of them are fast and some are not’ which is entailed.[6]

When a conditional statement is used within an argument it is like an inference within an inference. If the antecedent condition has not been met then the conditional must be considered hypothetical, which means that the argument as a whole has a hypothetical premise, and though it could be valid, it could not be considered sound. One could not prove a conclusion that is actual from a hypothetical premise. Relative to the actual world that premise is not true, or at least is not currently realized. For example:

If x is a unicorn then x is a one-horned creature
<u>x is a unicorn</u>
Therefore, x is a one-horned creature

Premise 1 is hypothetically entailed as a conditional statement, and the inference from premises to conclusion is valid. So, the argument is valid, but not sound. x could be a one-horned creature in reality, but not because of these reasons, as both of these premises are hypothetical. Suppose that we said instead:

If x is a one-horned creature then x is a unicorn
<u>x is a one-horned creature</u>
Therefore, x is a unicorn

In this case the argument is invalid because there are other things that are one-horned creatures besides unicorns, at least as unicorns are typically conceived. It may be strong or moderate (hypothetical) as an inference, but the conclusion would not be realized in the actual world. One could consider the conditional in the first premise to be not at all implied (actual) because there are things in the actual world that are one-horned creatures, but none of those things are unicorns. So that premise would be false in actuality as well.

Here is another example which has the form of hypothetical syllogism:

If A then B	If dogs were cats then dogs would be felines
<u>If B then C</u>	<u>If dogs were felines then they would be of the same genus as tigers</u>
If A then C	If dogs were cats then they would be of the same genus as tigers

[6] In using the term ‘entailed’ I do not mean that the consequent is a logically necessary consequence of the antecedent in the sense of concept containment or other types of necessity. I consider it entailed simply because it is not the case that all horses are fast, nor is it true that no horses are fast; it is the case that some horses are fast and some are not. The facts could have been otherwise, perhaps in some possible world they would be, but that is how it is in the actual one, so I consider the antecedent to entail the consequent (at least relative to the actual world), while ‘If there are horses then all of them would be fast’ or ‘If there are horses then none of them would be fast’ would be not at all implied. Perhaps one could consider this localized necessity or entailment (relative to the actual world) whereas inferences based upon concept containment would be universally entailed.

This is a valid inference, but the conditions ‘If dogs were cats’ and ‘If dogs were felines’ have not been met, so the conditional inferences and the argument as a whole are merely hypothetical.[7]

A classic example that is often used to demonstrate validity is:

All men are mortal
<u>Socrates is a man</u>
Socrates is mortal

One could also state it as:

All men are mortal
<u>If Socrates was real then he would be a man</u>
If Socrates was real then he would be mortal

In this case, a condition has been stated, ‘If Socrates was real’ and it has obviously been met, so the premise is actual. The argument is valid and sound, just like the prior one. It is not usually necessary to state a condition that has obviously been met, but it could be done. This becomes relevant in arguments with hypothetical premises, such as the following:

All men are mortal
<u>If the fictional character Hamlet was real then he would be a man</u>
If the fictional character Hamlet was real then he would be mortal

This argument is valid because the conclusion follows from the premises. However, it is not sound, nor would it be if the conclusion was ‘Hamlet is mortal’, because the condition stated in the second premise has not been met. Whenever a hypothetical premise is used the argument is hypothetical. (Hamlet is mortal in Shakespeare’s play, of course, but that would be a different context; here we are speaking of the actual world.)

If a conditional inference is only implied with a certain level of probability, say if the antecedent implies the consequent with a 70% likelihood, then when it is used as a premise the argument would have no more than a 70% likelihood of being sound.

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[7] Yet another oddity of the aforementioned truth table is that based upon it this argument would have to be considered not only valid but also sound. That is because if the antecedents of the conditional statements are false, then the whole conditional would be true, resulting in both premises being considered true, and the conclusion as well. It is valid because the conclusion cannot be false, and sound because the premises could not be false. In fact, any hypothetical syllogism with a valid form and conditionals with false antecedents would have to be considered both valid and sound. More broadly, any premise that is a conditional statement with a false antecedent would be true according to the truth table, which would lead to some very strange results concerning which arguments are considered sound or cogent.

Next I would like to discuss biconditionals. For a biconditional, the antecedent must be both a necessary and a sufficient condition for the consequent. When people use a conditional sometimes a biconditional is implied. For example, a parent might say ‘If you want cake you have to eat all of your vegetables.’ The parent has stated a necessary condition, and usually that is enough, because it is assumed that if the child meets the condition they will be given the cake; however, there is no actual guarantee that eating all of your vegetables will necessarily lead to getting a piece of cake, it is only guaranteed that if you do not eat all of them, you have no chance of getting cake. Perhaps if the parent is a lawyer looking for a loophole out of this informal contract they might remind the child of this after the vegetables are gone. The child would probably never forget the difference between necessary and sufficient conditions, but such injustice would likely lead to a howl that would split eardrums, and justifiably so. To avoid even the possibility of such chicanery, one could require that both conditions are made fully explicit as part of the deal by using a biconditional: ‘You will get cake if and only if you eat all of your vegetables’. This would make eating all of your vegetables both a necessary and a sufficient condition for getting the cake.

Biconditionals are interpreted by others to mean that both the antecedent and the consequent must always have the same truth value. This idea comes from the truth table for material equivalence, and it may even seem to be implied from the standpoint of necessary and sufficient conditions, for if the antecedent is both necessary and sufficient for the consequent then the consequent would have to be both sufficient and necessary for the antecedent, and they would always have the same truth value.

If the antecedent is both necessary and sufficient for the consequent (as it would be in a legitimate biconditional) then they would have the same truth value, but the antecedent is the condition, or in this case the bicondition. It is not the case that the consequent is a condition for the antecedent; that is more of a secondary implication of the original claim rather than part of the original claim. To suggest that the consequent equally implies the antecedent gives the impression that the two simple propositions are interchangeable, and actually they would be according to the truth table. But of course, ‘You can have vegetables if and only if you eat all of your cake’ obviously has a much different meaning than the original. I am sure that children everywhere wish that the two were equivalent, but they are not. We cannot switch which simple proposition is the proposed condition and which is supposed to be the consequence of that condition being met and have it be a logically equivalent claim. It would be like switching cause with effect, or a premise with the conclusion. When we say that the consequent is both sufficient and necessary for the antecedent it should be understood only as a secondary implication of the initial claim. It does not mean that eating cake (the consequent) is sufficient and necessary for eating vegetables (the antecedent), it would only be that if it is true that you are now eating, or have eaten some cake, that result is sufficient and necessary to know that the condition must have been met; in other words, it must be the case that you ate your vegetables (past tense), as, according to the original claim, there is no other way that you could have gotten cake. But the antecedent and the consequent are not interchangeable. ‘B if and only if A’ is no more equivalent to ‘A if and only if B’ than ‘If A then B’ would be equivalent to ‘If B then A’.

This is a subtle, but important distinction. The consequent is not a bicondition for the antecedent. ‘You will have eudaimonia if and only if you have virtue’ is not logically equivalent to ‘You will have virtue if and only if you have eudaimonia’. What is desired is eudaimonia; stating that virtue is a condition for it tells you what you must have or do in order to get what you want. In the original claim eudaimonia is the outcome; it is not a stated condition for anything, although if one did have eudaimonia, then it would be known that the condition of having virtue must have been satisfied.[8]

In light of these considerations, something needs to be done about the symbolization that is used for biconditionals. The triple bar symbol (≡) stands for ‘material equivalence’ and is often used in conjunction with the horseshoe symbol (⊃) that stands for material implication. Its truth table comes from simply conjoining the truth tables of two material conditionals. A ≡ B is considered logically equivalent to (A ⊃ B) • (B ⊃ A) and also equivalent to B ≡ A. The table says that whenever both simple propositions have the same truth value the biconditional is true, even if they have absolutely no relation to one another. It should be obvious by now why this analysis is flawed. The double arrow symbol (⟷) is also sometimes used to represent biconditionals, but this is problematic because it gives the false impression that the antecedent and the consequent equally imply each other.

The symbolization I would use is to have two lines, one above the other, with the arrow pointing in the same direction for both (=>), which indicates that the antecedent is both a necessary and a sufficient condition for the consequent. ‘If A then B’ would be symbolized as A⟶B, and ‘B if and only if A’ would be A=>B. What follows ‘if and only if’ is always the condition, and should be made the antecedent no matter what order it comes in the sentence. ‘You will have eudaimonia if and only if you are moral’ would be M=>E. ‘I will let you have some cake for dessert if and only if you eat all of your vegetables’ would be V=>C. These examples show how a biconditional is most commonly phrased, but it would not be incorrect to state the bicondition first, as in: ‘If and only if you eat your vegetables, then you will get cake for dessert’ which would still be symbolized the same way, V=>C.

There is some difficulty in translating into symbolic form the phrase ‘only if’. Should ‘A only if B’ be symbolized as A⟶B or B⟶A? B is the condition, and A is the result, so in some ways it seems more correct to say B⟶A. Would it be correct to say ‘Only if B then A’? B is clearly a necessary rather than a sufficient condition, but necessary conditions can be stated as the

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[8] The same holds for standard conditionals. If the antecedent is a sufficient condition for the consequent then it is implied that the consequent is necessary for the antecedent, but this is only a secondary derivative implication, not part of the original claim. We could say that if the consequent has not been realized then the antecedent must not have occurred, but this is simply reasoning that the condition must not have been met. That is not the same as saying that the consequent is a condition for the antecedent. The consequent is never really a condition that must be met in order for the antecedent to occur. The claim is always an inference from antecedent to consequent, never the other way around.

antecedent. However, this symbolization is problematic because ‘only if’ seems to have a somewhat different meaning than ‘if’. When a necessary condition follows ‘if’ there is a certain degree of probability that the consequent follows, resulting in a conditional that is strongly, moderately, or weakly implied. But ‘only if’ seems to indicate necessity rather than a probabilistic claim. Yet ‘If A then B’, or A⟶B does not really seem to fully capture the meaning of ‘A only if B’ either.

The claim ‘You will pass only if you study’ seems fairly equivalent to ‘You will not pass unless you study’ which in turn is somewhat equivalent to ‘You won’t pass if you don’t study’. A necessary condition must occur in order for the other event to be realized, so this is saying that if the necessary condition is not realized the other event will not be realized either. So ‘only if’ would be roughly equivalent in meaning to ‘unless . . . not’ or ‘if not . . . then not . . .’ The most correct symbolization of ‘You will pass only if you study’ would therefore be ~S⟶~P. This claim has necessity: ~S is enough to guarantee ~P. The contrapositive of this is P⟶S, or ‘If you pass then you study’, which is how others would symbolize it, but that is not entirely correct because passing was not really intended to be a condition for studying in the original claim. It would need to be something more like ‘If you passed then you must have studied’ (notice the past tense rather than future tense, which is not acknowledged in the symbolization) meaning that if you knew that P, or ~ ~P, was the case, then you would know that S, or ~ ~ S, was the case. This is a derivative claim that is implied by the original, but not the claim itself.

With this understanding of ‘only if’ the phrase ‘if and only if’ would not be equivalent to (A ⊃ B) • (B ⊃ A), which would actually be more like ‘if and if’; instead, it would be roughly equivalent in meaning to (A⟶B) • (~A⟶~B). V⟹C is approximately equivalent to ‘If you eat all of your vegetables then you will get cake for dessert, and if you do not eat all of your vegetables then you will not get cake for dessert,’ or (V⟶C) • (~V⟶~C). (Once again, one implies the other, but ~V⟶~C is not equivalent to C⟶V.)

Now some thoughts regarding proofs. One might anticipate, based upon what has already been said, that I would not agree with several of the rules of replacement, and that assumption would be correct. For example, the second formulation (the first has already been addressed) of material equivalence: (p ≡ q) :: [(p • q) v (~p • ~q)] is not actually equivalent to a biconditional at all. Suppose P represented ‘Plato was a philosopher in ancient Greece’ and Q represented ‘the sun has greater mass than the earth’. Well obviously since both of these statements are true, (~p • ~q) would not be correct, so it would have to be the other side of the ‘or’ statement which is correct, and indeed it is as a conjunction: ‘Plato was a philosopher in ancient Greece and the sun has greater mass than the earth’ is true. But it is most definitely not correct to say ‘The sun has greater mass than the earth if and only if Plato was a philosopher in ancient Greece’ because it is not the case that Plato being a philosopher in ancient Greece is a necessary or a sufficient condition (let alone both) for the sun having more mass than earth. Similarly, if P represented ‘Michelangelo painted the *Mona Lisa*’ and Q represented ‘Friedrich Nietzsche was a devout

Christian' then conjoining them as $(\sim p \bullet \sim q)$ would be correct, as both P and Q are false, but this does not mean that 'Friedrich Nietzsche was a devout Christian if and only if Michelangelo painted the *Mona Lisa*' would be a good inference from antecedent to consequent because there is no reason to think that Michelangelo painting the *Mona Lisa* would be necessary and sufficient for Nietzsche being a devout Christian.

Another of the rules of replacement is known as material implication: $(p \supset q) :: (\sim p \vee q)$, or 'If P then Q' is logically equivalent to 'Not-P or Q', and one may replace the other within the context of a proof. This is not at all surprising because the truth table for $(\sim p \vee q)$ shows it to be logically equivalent to $\sim(p \bullet \sim q)$. But of course I do not think that a conditional statement is really equivalent to either of these. For example, it is true that 'It is not the case that Istanbul is a city located in China and/or the earth revolves around the sun' but the city of Istanbul being located in China is most certainly not any sort of condition for the earth revolving around the sun. Many such examples could be given. This should not be considered a valid step in a proof.

The rule of exportation: $[(p \bullet q) \supset r]::[p \supset (q \supset r)]$ is incorrect as well. In the first case, P and Q together are a condition for R; in the second, P itself is a condition for $Q \supset R$. 'If Sandra and Jim go to the party, then Alex will go too' is different than 'If Sandra goes to the party, then if Jim goes Alex will go'. Suppose that Alex does not attend the party. What would we be able to infer from knowing that? In the first example we would be able to conclude that either Sandra or Jim did not attend, but we would not necessarily be able to say that neither one of them did. That may indeed be the case, but all that would be necessarily implied is that at least one of them did not attend, and we would not know which. In the second example, if Sandra goes to the party then the condition $J \longrightarrow A$ is triggered and Jim cannot attend without Alex, but there is no necessity that Jim must go to the party if Sandra does, it is only that if he attends Alex must as well, and even that is only in effect if Sandra attends. Just based upon this condition alone, it would also be possible for Alex to attend when Jim does not, which means that Alex could have gone alone, or he and Sandra could have both gone without Jim, but we have already been told that Alex did not attend, which eliminates those possibilities. So, if we know that Alex did not go, it is possible that none of the three went to the party, and it is also possible that Sandra went by herself. One thing that we know for sure, though, is that Jim must not have gone. I admit that this is a subtle difference but it shows that they are not logically equivalent.

Lastly, conditional proof is not valid in most of the cases in which it is used. According to this method, an assumption is made in a proof sequence, which is indented to show it is of a hypothetical nature, and tagged with the designation 'ACP', or 'assumption for conditional proof'. This will become the antecedent of the conditional statement that is meant to be proved. The consequent is derived through a series of valid steps using the rules of inference. Once the consequent has been obtained the conditional sequence is discharged on the next line (which is not indented) in a conditional statement. This line is tagged with the designation 'CP' meaning 'conditional proof' as its justification, together with the numbers of the first through the last lines of the conditional sequence. It is an elegant method, but what if the antecedent is not really a condition for the consequent? Often it would not be. You are allowed to assume literally anything for the antecedent, and as long as you can use the rules of inference to derive the consequent, that is supposedly all that is needed to show that the consequent necessarily follows from that initial assumption. But of course it is not: it has to be more than just a random pairing of claims to be a legitimate conditional. Moreover, if the initial assumption is false, then that line and the line containing the conditional could not be considered actual, so no actual conclusion would follow.

If a conditional statement that is used in an argument is considered to be an inference within an inference, one could simply make that inference from antecedent to consequent instead of trying to demonstrate it through a conditional proof sequence. One could not just assume anything for the antecedent though. If and only if A has already been demonstrated on a prior line of the proof, or is given in the premises, and A implies B, then one could infer on a new line 'If A then B' even if B has not been given or derived from an earlier line. The strength of that line would depend upon whether the antecedent entails the consequent, or how strongly it implies it. Only if it is actually entailed would it be unimpeachable. If the conditional is merely implied, then that line would only be true with the same level of probability as what the conditional inference has and the overall proof would not be valid.

This is different than when an implied conditional is used in the premises; in that case the probability of the conditional is a factor in whether that premise is considered true, or how often it would be, and so this affects the soundness of the argument rather than validity. But here it is validity because the steps within the proof sequence are intended to show how one could derive the conclusion from the premises. If any one of those steps from premises to conclusion has less than 100% likelihood then the proof is not valid. However it could still be strong or moderate.

In the following proofs P represents 'It is raining', Q represents 'the streets are wet' and S represents 'the buildings are wet'.

1. If P then Q		
2. P	// S	
3. Q		MP, 1, 2
4. If Q then S		CI, 3
5. Therefore, S		MP, 3, 4

'CI' stands for 'conditional inference'.

This would be an indirect proof in which the premises imply the stated conclusion rather than being directly derived from them, as Q would be. This conclusion is more of a secondary implication of the premises. A number of things could be 'proven' in this way because often it is the case that many things could be inferred indirectly from a set of premises besides the conclusion that is directly derivable from them. But such proofs would not usually be as strong as those in which one can directly prove the conclusion from the premises. The proof itself cannot be any stronger than the conditional inference. This proof, as stated, would be strong rather than valid because the conditional inference is very strongly implied rather than entailed. However, it could easily be modified so that it was valid if the conditional inference was ‘If P then S’ rather than ‘If Q then S’. This would be a perfectly legitimate step in the proof because P is a premise of the argument and ‘If it is raining then the buildings are wet’ is practically entailed.

1. If P then Q		
2. P	// S	
3. If P then S		CI, 2
4. Therefore, S		MP, 2, 3

Here is one more example:

1. If P then Q		
2. ~Q	// ~S	
3. ~P		MT, 1, 2
4. If ~P then ~S		CI, 3
5. Therefore, ~S		MP, 3, 4

This proof would also be very strong rather than valid. The conditional inference is ‘If it is not raining then it is not the case that the buildings are wet’, which is very strongly implied, but not entailed, because it is possible (though unlikely) that the buildings could be wet for some other reason. This proof could be modified so that the conditional inference was ‘If ~Q then ~S’, but it would still not be valid; in fact, it is actually a little stronger in its current form, though it would be quite strong either way.

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