Existential Import

By David Johnson

A proposition is a claim that is either true or false. A categorical proposition makes a claim about the connection, or lack of it, between two categories. There are four standard forms of categorical propositions which express how the subject category (S) is related to the predicate category (P):

A: All S are P
E: No S are P
I: Some S are P
O: Some S are not P

A asserts that the entire subject class is included in the predicate class, and E claims that no members of the subject class are included in the predicate class. These claims are universal because they are about every member of the subject category. The I statement claims that at least part of the subject category is also a member of the predicate category, and O asserts that at least part of the subject category is not included in the predicate category. These are particular claims because they are not necessarily about every member of the subject class.

In the A statement ‘All horses are mammals’ the subject is ‘horses’ and the predicate is ‘mammals’. The statement is asserting that all members of the ‘horses’ category are also members of the ‘mammals’ category. This is obviously true because ‘horses’ is a species within the class of *mammalia*, which means that it is a subcategory of the larger ‘mammals’ category, and is entirely contained within it. However, there has been more controversy over similar kinds of categorical statements, such as ‘All unicorns are one-horned creatures’. It seems as though it must be considered true, but of course there is no such thing as unicorns, so if the proposition is asserting, or even implying that unicorns actually exist, then perhaps it should be considered false. Whether a categorical proposition implies the existence of one or more members of the subject class, or in other words whether it has ‘existential import’, or makes an ‘existential assumption’, is the primary consideration in determining its truth value for what has come to be known as the ‘Boolean interpretation’. George Boole, in *The Mathematical Analysis of Logic* wanted to show how logical expressions could be rendered as algebraic equations. Here is how he rendered the four categorical propositions:

A: $x(1 - y) = 0$ equivalent to ‘All x are y’ or ‘No x are non-y’
E: $xy = 0$ equivalent to ‘No x are y’ or ‘No y are x’
I: $v = xy$ equivalent to ‘Some x are y’ and/or ‘Some y are x’
O: $v = x(1 - y)$ equivalent to ‘Some x are non-y’ and/or ‘Some non-y are x’

In this interpretation, x and y represent classes, and 1 - y represents the universe with the y class subtracted out. Boole says that what he means by 'universe' is 'every conceivable class of objects whether actually existing or not.' I might have thought that he would have chosen the infinity symbol to represent this, but I suppose it is better to use '1' instead because it is unknown whether there would be an infinite number of classes. At any rate, 1 essentially means everything, or all conceivable classes, and 0 means nothing. 1 - y could be thought of as equivalent to non-y, or everything except the y class. So this first equation is saying that the product of x and non-y is zero, or has zero members. Thus, no members of the x category are members of the non-Y category, or 'No x are non-y', and this is equivalent, by obversion, to 'All x are y'. The second equation, xy = 0, means that the product or intersection of the x and the y category is zero, or there is nothing which is both an x and a y at the same time.

The third and fourth equations use the symbol v which Boole defined as meaning 'some'. The third equation means that the intersection or product of the x and the y class has some members.[1] Therefore we can say 'Some x are y' (and 'Some y are x'). This would mean that at least one member of the universe of conceivable classes (some is considered to mean at least one) is both an x and a y. The fourth equation, v = x(1 - y), says that the product or intersection of x and non-y has some members. There is at least one member of the x category that is also a member of the non-y category, so 'Some x are non-y'; or, equivalently (through obversion), it also means that 'Some x are not y'. One could also say, either through conversion, or merely from the equation, that there is a non-y that is an x.

I do not really have a problem with what has been said so far, or at least not much of one. Boole's rendering of the categorical propositions, as described above, seems perfectly legitimate. The problematic aspect comes from how John Venn interpreted, or I would argue, misinterpreted Boole. Venn defended Boole's mathematical interpretation (or what he thought was Boole's interpretation) from objections, but in the process, he made some claims of his own that I believe to be erroneous. What is often referred to as the 'Boolean interpretation' is not really Boole's position at all, it is Venn's, and from here on, I will be referring to it as such.

Venn interpreted universal propositions, such as 'All S are P' and 'No S are P', as claims that did not have any existential import; in other words, neither of them were necessarily making any claims that the categories they referred to had any actual members. Instead, they were only equivalent to a conditional statement, such as 'If something is an S, then it is also a P', or 'If there are any S, then they are not P'. The conditional nature of the claim means that it is not necessarily saying that there are members of S. Venn thought that the only unconditional commitment of 'All S are P' is that there are no members of the S class that are not members of the P class. This condition could be met even if there are no members of the S class at all. For the unicorn example this means that the statement is only committed to saying that if there is a

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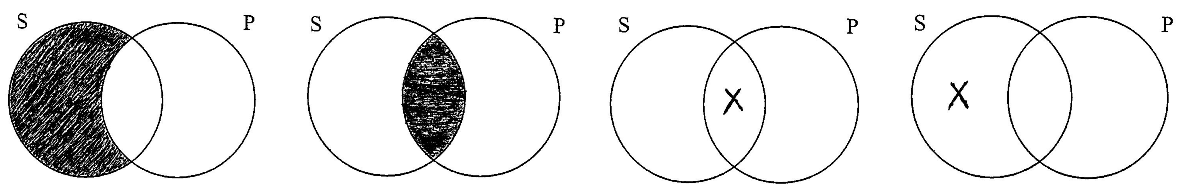
[1] Boole also used vx = vy to represent the I statement. He seems to have meant by this that there is an indeterminate amount of both x and y. It would be something like 'Some indefinite amount of x is some indefinite amount of y.'

unicorn, then it would have one horn, not necessarily that there are any unicorns. Because of this, Venn considered an A statement about unicorns to be true.

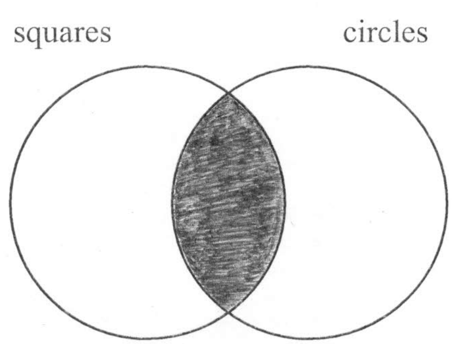
However, Venn interpreted particular statements differently. He thought that since ‘some’ is interpreted to mean ‘at least one’, particular claims must necessarily imply that at least one S exists. Thus, particular claims always have existential import. ‘Some S are P’ was interpreted (and is so today, following Venn) as ‘There exists at least one S and it is a P’, and ‘Some S are not P’ is interpreted as ‘There exists at least one S and it is not a P’. So, ‘Some unicorns are fast’ necessarily implies that at least one unicorn really exists, and would be false because of that.

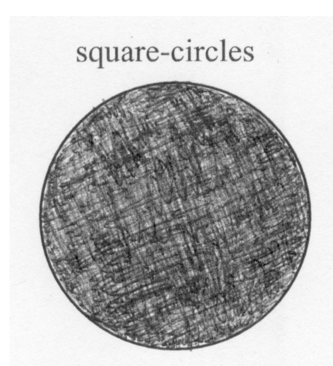
To understand Venn’s interpretation, it is helpful to refer to the diagrams that bear his name. Here is how the four categorical propositions are expressed using Venn Diagrams:

A: All S are P E: No S are P I: Some S are P O: Some S are not P

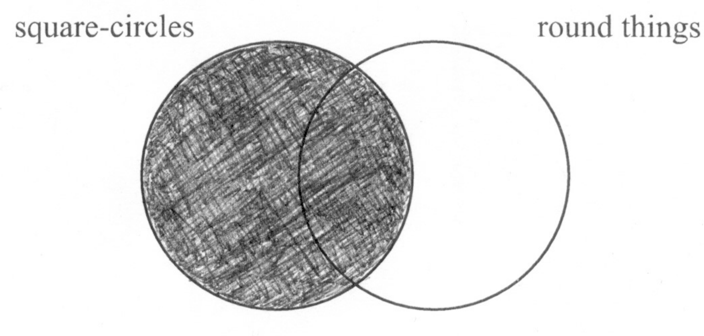


If you had two categories, one being squares and the other circles, the correct way to diagram it would be to shade out the area where the two categories overlap, which means, of course, that they do not overlap. This is equivalent to the E statement ‘No squares are circles’ (or ‘No circles are squares’). Shading an area means that it is empty. This correctly indicates that there is nothing that is both a square and a circle. This shaded area could also be represented as its own category, with of course, no members, as indicated below:





Now suppose that this category was used as the subject term in a new proposition, such as 'All square-circles are round':



For Venn, 'All x is y' simply means that there are no members of the x class that are outside of the y class, and nothing more is implied. This diagram indicates that no members of the 'square-circles' category are outside of the 'round things' category, because all parts of the circle outside of the 'round things' category is entirely shaded out, just as it is for any standard A proposition. In this case, the entire circle is shaded, but if the claim has no existential import, it does not matter; as long as the portion of the subject category that does not overlap the predicate category is shaded the claim is considered true. Similarly, 'No x is y' just means that there are no members of the x class that are inside the y circle, and nothing more is implied. Since all parts of the 'square-circles' category that is inside of the 'round things' circle is shaded, 'No square-circles are round' is also considered true.

The diagram indicates that the claim 'Some square-circles are round' is false because the area where the two categories overlap is shaded out, which means that nothing can be in there. This is the opposite of what the I claim says. 'Some square-circles are not round' would also be false, because once again, the area where the x is supposed to be is entirely shaded.

Both universal statements would be considered true, and both particulars would be considered false whenever the claims refer to an empty subject category, in which case the entire circle representing the subject category would be shaded. Never mind that they are contraries and subcontraries: to say that 'All S are P' and 'No S are P' are both true at the same time is like saying that something is both completely white and completely black simultaneously. One textbook author uses the example of 'All shoplifters are prosecuted' and 'No shoplifters are prosecuted' to explain and defend this view.[2] Following Venn, he says that if there are no shoplifters then no shoplifters are *not* prosecuted, so the claim 'All shoplifters are prosecuted' is true, and no shoplifters are, or have been prosecuted, so the claim 'No shoplifters are prosecuted' is also true. I disagree. It is true that neither claim has been, or even could be falsified unless there is a shoplifter, but they could not be verified without one either. Suppose that someone challenged the claim that 'All shoplifters are prosecuted' is true. What kind of

[2] Johnson, Robert M. *A Logic Book: Fundamentals of Reasoning.* 5th Ed.

evidence could you produce to prove it? No one knows with certainty what would happen once there is a shoplifter. What if the shoplifter was the son of the mayor? Can you say for sure that he will be prosecuted? On the other hand, could you really say that 'No shoplifters are prosecuted' is true either? There is no evidence to believe one way or the other. For all we know, perhaps neither one would be true, and if there are shoplifters it will actually be the case that some of them are prosecuted and some are not. There has to be at least one shoplifter before the actual truth value of any of these claims can be known, so until then, all four have an undefined truth value. More will be said of this below.

Now one could consistently say 'No shoplifters *have been* prosecuted', and 'All shoplifters *will be* prosecuted', the latter being how most signs actually read. Those claims can potentially both be true at once because they refer to different time frames. Instead of saying the equivalent of 'I am running and I am kneeling', it would be like saying 'I have been running, but I will soon be kneeling', or vice versa, and there is nothing problematic about that.

This interpretation of categorical logic yields some extremely odd results. It effectively means that any universal claim that one might make about a category that has no actual members is always true, no matter how silly, bizarre, or even self-contradictory it might be. 'All unicorns have three horns', 'No unicorns are one-horned creatures' and, 'All mermaids are male' would all be considered true. Each would be diagrammed the same way, by shading out the entire circle for the subject category, just as we did for 'square-circles', and the same reasoning would apply for why they are considered true. Speaking of square-circles, they must be triangular as well, because according to Venn, 'All square-circles are triangles' is also true. I wonder what type of triangle a square-circle would be classified under? Isosceles, perhaps? Or would it be an equilateral?

Venn thought that for universal claims neither S nor P had to have members, so in 'All unicorns are dragons' both circles would be entirely shaded out. Since it is true that there are no unicorns outside of the dragons category (because there are no unicorns at all) the claim would be true. But does it seem to you that 'If it is a unicorn, then it is a dragon' should be considered true, even as a conditional statement?[3] I have a hard time seeing how being a unicorn would be a legitimate condition for being a dragon, or what would be the rationale for saying that it was. Even 'All leprechauns are both short and not short' would be considered true. (Once again, both the 'leprechauns' category, and the category of 'things that are both short and not short at the same time' would be entirely shaded.)

Another rather strange part of this interpretation is that if you have a subject category that does have actual members, and a predicate category that does not, such as 'All horses are unicorns' then both circles would end up being entirely shaded even though there are horses. This is

[3] For material conditionals, whenever the antecedent is false, the entire claim is considered true. Under that interpretation, this conditional would be true, but as one might expect, I do not agree with that interpretation of conditional reasoning at all. However, my argument against it will have to wait for a separate essay.

because the ‘unicorns’ category must be completely shaded already, being empty, and according to how A statements are supposed to be diagrammed, all parts of the ‘horses’ category outside of the ‘unicorns’ category must also be shaded. Thus, strangely, the diagram would indicate that there are no horses or unicorns. Now I suppose that one could just say that this shows the claim to be false, because since we know that there are horses, we could put an x in the area of the ‘horses’ circle that does not overlap the unicorns circle, and then we would know that it was wrong to shade that area. But the whole point of Venn’s analysis was supposed to be that a universal statement does not necessarily make any existential claims. If it is really true that the statement does not have any existential import, then we cannot do that. We must assume that the ‘horses’ category may or may not have members, and based upon that assumption, the diagram shows that there are no horses or unicorns, and the claim must be considered true. Anytime the predicate category is empty the A statement would always be true, no matter what the subject category is. Apparently all members of every other category are also unicorns, including you and me.

There are other universal statements that, while maybe not directly contradictory, still should not be considered true because they are inconsistent with our knowledge of the subject, such as ‘All dragons are cows’, ‘No dragons breathe fire’, and ‘All mermaids live in desert climates’. Each of these claims would have a subject class that is entirely shaded out, which means that literally any universal claim about that subject would always be considered true, whatever the predicate. ‘All dragons are reptiles’, ‘All dragons are insects’, ‘All dragons are mammals’, ‘All dragons are humans’, ‘All dragons are trees’, etc. All would be considered true, which is truly absurd. A term can still have meaning even if it does not refer to an actual object. There are no actual cyclops, but ‘All cyclops are two-eyed creatures’ cannot be considered true because the term ‘cyclops’ comes from Greek mythology, and this claim is not true of the class of fictional objects that the term refers to in the myths.

Equally peculiar, any particular claim about nonexistent subjects is always considered false because of existential import, even if the subject necessarily entails the predicate, as in: ‘Some unicorns are one-horned creatures’, or ‘Some mermaids are female’ etc. Other claims are not necessarily true by definition, but their truth is strongly implied by our knowledge of the subject, such as: ‘Some mermaids are aquatic’, ‘Some dragons are not cows’, and so forth.

How about ‘Some A are A’? Suppose that ‘A’ stands for a category that does not have any members with actual existence. The claim is supposed to be false simply because of existential import? And ‘No A are A’ is supposedly true? How could that be? If A = A then it cannot be the case that no A are A. The principle of identity holds as much for fictional subjects as it does for actual ones. I say that ‘No unicorns are unicorns’ must be false, and ‘All unicorns are unicorns’ has to be true, according to the principle of identity, whether or not there are any actual unicorns. Similarly, ‘All mermaids are non-mermaids’ (‘non-mermaids’ would include everything except mermaids) has to be false, because it is self-contradictory. It should never be the case that ‘All A are non-A’ (or ‘All non-A are A’) is considered true, no matter what ‘A’ stands for. A coherent logical system does not produce self-contradictory results.

Venn believed that you cannot make the claim ‘Some S are P’ without implying that at least one S actually exists. Well, why not? If the universals are supposed to be equivalent to a conditional statement then why not the particulars? You could say ‘If there are any S, then at least one of them is P’ and ‘If there are any S, then at least one of those is not a P’. There is no reason to treat universals and particulars differently. It is an arbitrary unwarranted distinction. In reality all four claims must have the same existential commitment because they all refer to the same categories.

Here is how Aristotle originally formulated the categorical propositions:

A: Every A is B
E: No A is B
I: Some A is B
O: Not every A is B

The differences between Aristotle’s formulations and those used today are relatively subtle. However, I believe that the original formulations show the intended relations a little more clearly. The biggest difference is with the O statement. The question is whether the ‘not’ should refer to ‘A’ only, or to the entire claim. The most precise logical opposite of ‘Every A is B’ is ~ (Every A is B), or ‘It is not the case that every A is B’. I would argue that this rendering more accurately captures the meaning of the O statement as the contradictory of A. But of course this does not imply the existence of the subject any more or any less than the A claim does. Under Venn’s interpretation it would be equivalent to ‘If there are any A then not all of them would be B’. If ‘A’ is an empty class, then ‘Not every A is B’ would have to be considered true just like ‘Every A is B’ and ‘No A is B’. This suggests that the I statement carries the same implication, as simply the negation of E. ‘Some A is B’ is probably just a simplification of ‘Not “No A is B”’. Just as in mathematics, you can simplify a double negation to a positive claim, as in - (-2) to +2, so ‘It is not the case that no A is B’, or ~ (No A is B) can be simplified to ‘Some A is B’. But I do not think that this necessarily commits one to saying that there is at least one actual member of the subject class any more or any less than the E statement does. It would be equivalent to ‘If there are any A, then it would not be the case that no A is B’. If we are going to say that A and E are always true whenever the subject class is empty then really all four claims would have to be considered true.

One could perhaps derive Venn’s system from Boole’s if one understood Boole to mean by ‘universe’ only objects with actual existence. That may have been how Venn interpreted him, but that is clearly not what Boole meant. Boole argued that an A statement allows for conversion by limitation, which means that he thinks ‘All Xs are Ys’ implies ‘Some Ys are Xs’ and ‘No Xs are Ys’ implies ‘Some not Ys are Xs’. So Boole would think that if ‘All dogs are mammals’ is true, then ‘Some mammals are dogs’ must also be true. If universal propositions imply particular propositions for Boole, that indicates that he was assuming that the classes were not empty.[4] One could interpret this to mean that Boole thought that universal propositions would have existential import, but I do not believe that he was even considering the issue of existential import - that was Venn's thing. Recall that Boole defined the 'universe' as 'every conceivable class of objects *whether actually existing or not*' (emphasis added). He was only assuming that the categories had members of some kind. Boole would *not* have said that a category was empty if it had no members with actual existence but did have hypothetical or fictional members. A particular claim does imply that there is at least one member of the categories referred to, but it would not necessarily mean for Boole that those members have actual existence in the real world.

It is apparent from the text of Venn's work *Symbolic Logic* that he thought of himself as having a distinct system that went beyond Boole: for example, on page XXV of the introduction he says 'the only unconditional implication of even an affirmative universal categorical proposition is to be found in what it denies . . . In this systematic form the interpretation is, I believe, novel . . .' Venn probably thought that Boole's interpretation implied his own, and that is why he defended it, but I would disagree.

This brings up another point of contention. I disagree with the quote stated above. It is not the case that the only thing that is unconditionally implied by a universal statement is what it denies; that is part of what it implies, but there is a positive aspect to the claim as well. Venn thought that only the negative implication would always be in effect because if the categories were empty - and he thought that they could be because universal claims do not have existential import - there would be no positive implication, even though it is an *affirmative* statement. But here we need to make a distinction between categories that have no members with actual existence and those that have no members of any kind. A categorical proposition, whether it be universal or particular, would make no sense if the subject category is entirely devoid of members. A category or class is not a thing, it is a group of things; if it has no members then there is nothing at all. That would be like an alphabet without any letters, or a team with no players. 'All [nothing] are mammals' is not a coherent claim. Nothing is predicated of 'nothing'. It does not make any sense to refer to 'nothing' as though it were something. Any claims about the supposed attributes and characteristics of nonexistent members of an empty category, or the other categories that those nonexistent members would, or would not belong to, do not have a truth value, they are just undefined.

Some say that the E statement could be true if one or both of the categories are empty. The argument is that 'No S are P' simply means that there are two distinct categories that never overlap, meaning that there is nothing which is a member of both. If one or both of those categories are empty, then, so the argument goes, obviously there is nothing that is a member of both, so the claim would be true. But if one of the categories has no members, then really there is only one category (and if neither does, then there is nothing at all). We cannot really affirm that

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[4] Patrick Hurley makes this point in *Existential Import: Historical Background* a supplement to the textbook *A Concise Introduction to Logic*, 12th edition.

‘nothing’ does *not* have a certain predicate any more than we can affirm that it does have one. You cannot make any definitive claims about the predicates of ‘nothing’, including negative ones, because there is no thing to make a claim about.[5]

As an analogy, division by zero is impossible in ordinary arithmetic. A fraction with a denominator of zero, such as 4/0 is undefined. I am by no means an expert in mathematics, but my understanding of the reason why is that as the denominator gets smaller and approaches zero, the numerator must approach infinity, and it would be absurd for a finite number to be infinite, by definition. The first step in understanding this is to remember that every real number has 1 as a denominator, whether it is displayed or not. Anything multiplied by 1, or divided by 1, is simply that number, which is obvious if you think about it. 4 multiplied by 1 (meaning that I have 1 set of 4) is still 4, and if I have 4 apples, and they are divided one way, then I still have 4 apples. Notice the symmetry between multiplication and division, being the inverses of one another. As the denominator of a fraction gets smaller, the numerator gets larger. 4/.1 = 40, 4/.01= 400, 4/.001= 4,000, etc. And, of course 40 x .1 is 4, 400 x .01 = 4, 4000 x .001 = 4, etc. So, the closer that the denominator of a fraction gets to zero, the closer the numerator gets to infinity. But infinity is outside the set of real numbers, or maybe a more precise way to put it is that it *is* the set, the entire thing. The infinity symbol does not stand for a specific number, it represents an open-ended set with an unlimited quantity. A fraction like 4/.000000000000001 is seldom used, but it is still defined within the set of real numbers, being equivalent to 4,000,000,000,000,000 (the numerator is this many times larger than the denominator). However, 4/0 is undefined, because if zero is the denominator then the numerator would have to be infinite, and yet you are also saying that the numerator is 4. Not only is it outside the set of possible values, it is saying something that is self-contradictory, so it is an undefined expression. If 4/0 did have a defined value, and the inverse relation between multiplication and division holds, then that would mean that 0 x ? = 4 must have a specific value. There would have to be some actual number that if plugged into this equation would make it true. But zero times any number is zero.

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[5] A special problem arises when existence itself is the predicate. Of course, Immanuel Kant would say that existence is not really a predicate at all, but I am not sure that is true. How would one translate claims from propositional logic such as ‘There are no unicorns’, or ‘Unicorns do not exist’ into categorical form? If we were to consider them equivalent to ‘No unicorns are things that exist’ that would be undefined. But of course the propositional claims are not undefined, they are true. They obviously do not presuppose the existence of unicorns, as that is the very thing that they are meant to deny. What’s more, stated as a hypothetical, ‘If unicorns really existed then none of them would have actual existence’ is of course self-contradictory, so it would be false. I think the best translation would be ‘No things that exist are unicorns’. A subject that does not exist cannot have any properties, but the category ‘things that exist’ does have members, and one could accurately say that none of those members have the property of being a unicorn. Thus the claim has an actual truth value which matches the truth value of the propositional claims. (As will be further explained below, I recognize conversion as only conditionally valid if the categories have members, so ‘No unicorns are things that exist’ and ‘No things that exist are unicorns’ are not equivalent. The first is undefined while the second is true.) To say that something does exist, such as ‘There are dogs’ it would best be translated into categorical form as ‘Some things that exist are dogs’ and ‘Some things that exist are not dogs’. Obviously it would be wrong to say ‘All things that exist are dogs’ so the two particular statements would be true rather than a particular and a universal. Since both categories have members one could also say ‘Some dogs are existing things’.

There is no value that if multiplied by zero would equal four. For all of these reasons, it is best to consider 4/0, or any number divided by zero, undefined.

The way that many teachers assign a grade is to take the number of points earned by the student and divide that number by the number possible for that assignment. This gives a decimal point answer, which can then be multiplied by 100 to give a percentage score. These fractions (points earned divided by points possible) would usually have numerators that are smaller than the denominators, but if the student received extra credit then the number of points earned could be greater than the number possible, and the percentage score would exceed 100%. Imagine that for some class there was only one point possible for the entire semester. That would be unusual, but there is nothing contradictory about it. If the students did the assignment, and the work was satisfactory, they would earn the point, and their grade would be 1/1, or 100%; if not, then their grade would be 0/1, or zero percent. A zero can be in the numerator without any problems. It simply indicates that the student earned no points out of the number possible, which is a clearly defined score. But suppose that the total number of points possible for the semester was zero. How could one then say what would be a fair grade? With no points possible, there is nothing to base that grade upon. Any grade would be just as applicable as any other. No actual grade or percentage score could be assigned with any certainty. An A, B, C, D, or F would all seem to be equally correct, and there would be no way to distinguish which one it should be over the others. One could not even give a pass/fail grade because it would be impossible to tell which it should be. Having zero points possible is unworkable because the domain of possible values is set at nothing. That really means that every grade is possible and no grade is actual because the domain of possible values has not been clearly defined.

I believe that something similar is true with categorical propositions. An empty subject class could have an unlimited number of possible predicates, because there would be no way to invalidate any of them with certainty, but no actual ones. You cannot confirm or refute the claim that a subject has or lacks a certain predicate unless there is a subject. All four of the categorical claims tacitly assume that one is referring to something rather than to nothing. In math, they stipulate that the denominator of a fraction cannot be equal to zero. We ought to do the same with categorical propositions and stipulate that the subject category cannot be empty or it is an undefined expression.

However, one could make a hypothetical claim about a theoretical or fictional subject. If taken as a claim about the actual world, the category is empty, because it does not have any members with actual existence, and so it is undefined from that standpoint. But, depending upon the subject, sometimes we can tell what the hypothetical truth value would be. The most obvious candidates for this are analytic claims. Venn's interpretation would suggest that 'All bachelors are female' is false as long as there are actual bachelors, but if there were not any, then it would be true. Really, though, logical necessity is a function of language, not things. It is known *a priori,* merely from the definition of 'bachelor' that a bachelor must be male, regardless of whether there are any. Immanuel Kant referred to this as 'concept containment'. A claim can be entailed or implied by the definition (or known to be self-contradictory) whether the subject exists in the actual world or not.

‘All dragons are mammals’ may not necessarily contradict the definition of ‘dragon’ (though it could for some definitions), but at the very least, it is inconsistent with the connotations associated with the term. None of the fictional accounts of dragons represent them as mammals. A fictional subject has no actual predicates, so a claim that it has or lacks a certain predicate can only be hypothetical. As stated above, the claim is undefined. But the fictional accounts imply that the claim would be false if there were dragons. So the claim ‘If dragons actually existed, then all of them would be mammals’ is false. ‘Some dragons can fly’ is undefined as stated, but it would be true if there were dragons, or it could be considered true relative to the context of fantasy stories.

Now some might argue that ‘All unicorns are one-horned creatures’[6] must be true for the real world as well, not just fiction. But it does not refer to anything in the real world. From that standpoint it is equivalent to ‘All [nothing] are one-horned creatures’. A categorical proposition has to refer to some actual object in order for it to have an actual truth value. Any speculation about the predicates that a hypothetical subject would, or would not have, are merely hypothetical conjectures that cannot in actuality be verified or falsified. Even for an analytic claim that is entailed by the definition of the subject all that could be given is a hypothetical truth value. This is because if there is no subject then there are no predicates, so if the class is empty all that we can really say is that if there was a subject, then it would necessarily have that predicate. Although, if the claim was worded differently, it could have an actual truth value. ‘A unicorn is defined as a one-horned creature’ or ‘In fantasy literature a unicorn is depicted as a one-horned, equine-like creature’ or ‘According to ancient legend,[7] a unicorn is a one-horned creature’ are all true for the actual world. In these examples the claim is about the definition or the stories about unicorns (which do exist in the real world) rather than unicorns themselves, and the claim is accurate, so they are true. It just has to be stipulated in some way that the claim is not meant to refer to some object in the actual world that does not exist. If the object does not exist that does not necessarily make the claim false, it just means that it is not applicable.

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[6] Suppose that there was a unicorn that did not have a horn, either due to physical deformity (i.e. it was born that way), or because it had been broken off, or cut off, would the animal still be a ‘unicorn’? Is ‘unicorn’ the name of a hypothetical species, or is it meant as a literal description? If the former, then it would be. It is likely that the species was given that name because having one horn was its most distinguishing feature. But this would just be a general characteristic. There could still be a few outlier examples without the name being misapplied to the species as a whole. This shows that there can sometimes be exceptions even to analytic claims because they are based upon definitions, and definitions are often just based upon generalizations.

[7] I did not say ‘myth’ because unicorns do not actually appear in ancient Greek mythology (Pegasus had wings but no horn), or any other society’s mythology. Anciently they were believed to actually exist somewhere in the real world. It seems incredible that such a thing could have ever been believed, but we have to remember that anciently there were still many places in the world that had not been fully explored, so it probably seemed much more plausible back then.

‘All necessary existents exist’ is thought by some to be an analytic claim that must be true because to deny it would result in a contradiction. It is analytic, but it is possible that the subject term may only refer to something that is fictional. It may be contradictory to say that the claim is false, but it is not contradictory to say that it is undefined if there is no subject in the actual world. Even if existence would be an essential property for such a being, it is not an actual property unless the subject is actual. ‘If it is a necessary existent then it would exist’ is clearly true, but without knowing whether there is an actual member of the subject category, we cannot know whether the antecedent condition has been met; if it has, then the claim is actually true, because the antecedent is true, and the consequent follows from it with necessity; but if the antecedent condition has not been met, then the claim is only hypothetically true. It is known *a priori* that the subject would have that predicate, if there is such a thing, but it is not known *a priori* that there is a subject; that would need to be verified through experience. Thus, the claim that the statement is true is *a priori*, the claim that it represents a true state of affairs in the actual world is *a posteriori*.

Some claims could be hypothetically probable or improbable. For example, ‘If there were dragons then all dragons would be reptiles’ might be a reasonable assumption. It is not certain, because for all we know, if dragons really existed maybe they would be part of an entirely different genus than anything that exists in the real world right now. But, based upon the depictions and descriptions of dragons, it seems reasonable to say that the claim would likely be true if dragons existed. Therefore it should be considered true or probable.

One of the uses of conditional statements is to present counterfactual scenarios and to make claims based upon those hypothetical conditions. One could state the categorical propositions in conditional form, but I would do it differently than how Venn does. The correct form is ‘All x would be y if there were x’ and ‘No x would be y if there were x’. There is no reason why particular statements cannot be considered hypothetically as well, as in ‘Some x would be y if there were x’ and ‘Some x would not be y if there were x’. Here is an example:

A: ‘Every unicorn would be a one-horned creature, if there were unicorns’ True
E: ‘No unicorns would be one-horned creatures, if there were unicorns’ False
I: ‘Some unicorns would be one-horned creatures, if there were unicorns’ True
O: ‘Not every unicorn would be a one-horned creature, if there were unicorns.’ False

All of these claims are obviously counterfactuals because the condition ‘if there were unicorns’ has not been met. However, as counterfactual statements, it is obvious that A and I are true, while E and O are false. I consider this to be a defined truth value as a conditional statement whereas the equivalent categorical proposition alone has an undefined truth value for the actual world. For many fictional subjects, particularly those that have analytic claims associated with them, one could use the conditional statements above in the Aristotelian Square of Opposition and all of the same relationships would hold as for actual subjects. However, since they are counterfactuals, we would say that while the inferences of the Square are valid, they are not actual. This would be the difference between inferences about bachelors and those about unicorns: the former would be valid and actual, while the latter would be valid but hypothetical.

In the way that I have formulated them, the existence of the subject is the antecedent, and the claim is that this would be sufficient to guarantee the consequent, which is the entire categorical proposition.[8] This should produce the same truth value as if you were considering the categorical proposition only, the difference would just be that if the antecedent condition has not been met then the truth value is hypothetical rather than actual. All categorical propositions could be stated this way, but it is unnecessarily redundant to state the condition if it has very obviously been met. It would be rather silly to say, for example, 'If there were any dogs then they would all be mammals' when it is so obvious that there are dogs. Technically it is not incorrect, just unnecessary. I would only state the claim as a conditional when one (or both) of the categories does not have members with actual existence, or there is some uncertainty about whether it does.

Sometimes there is not enough information available about fictional or hypothetical subjects to know whether the claim being made would be true or false. For example:

1. 'All aliens have 2 eyes'
2. 'All aliens have 4 eyes'
3. 'All aliens have 8 eyes'
4. 'All aliens have 16 eyes'
5. 'All aliens have an infinite number of eyes'

Venn's interpretation is that all of these claims are true (along with an infinite number of similar ones) because, as far as we know right now, there are no aliens. Of course, if they were particular statements, such as 'Some aliens have 2 eyes' then he would say that all of them were false. But since universal claims are not considered to have existential import, if there are no aliens then it is true that there are no aliens outside of any of these predicate classes, so all of the claims would be considered true. Each would only be equivalent to a conditional statement, such as 'If it is an

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[8] One potential difficulty with this might be with categorical propositions that are merely probable. It seems as though the antecedent must be considered a sufficient condition (it is certainly not a necessary condition), but what if the consequent is not actually guaranteed to be true? Does that mean that the antecedent is not really sufficient to guarantee it? Actually yes, it does. We have to remember that just because something is claimed to be a necessary or sufficient condition does not mean that it really is one. In the statement 'If I toss a fair coin then it will come up heads' the antecedent is claimed to be a sufficient condition, but of course the consequent is not really guaranteed. It only has a 50% probability of being true if the antecedent is. I believe that a conditional's truth value depends upon just how necessary a necessary condition really is, and how sufficient a claimed sufficient condition is. This could be anywhere from 100% for analytic claims that are true by definition, 0% for claims that would be self-contradictory, and everything in between for claims that are not analytic. The conditional's truth value is a measure of the likelihood that the purported relation holds, or the frequency with which it does. The claim above would not really be true or false, it would be of moderate strength, with a 50% likelihood. A claim such as 'If there were dragons, then they would all be reptiles' should be considered highly probable rather than true or false. Perhaps the way that it could be stated is to say that it is highly probable that the antecedent condition would be sufficient to guarantee the consequent.

alien then it has x eyes', and according to the truth table for conditionals, whenever the antecedent is false (and it would be if there are no aliens), then the whole statement is always true, regardless of what the consequent says.

Even stated conditionally, 'If x is an alien, then x has 4 eyes' and 'If x is an alien, then x has 8 eyes' are still mutually exclusive claims about x.[9] It is possible that they could both be false (if for instance an alien actually had no eyes) but they cannot both be true at the same time; in other words, they are contraries. If it is true that 'All aliens have 4 eyes' then it must also be true that 'No aliens have 8 eyes' so the statement 'All aliens have 8 eyes' has to be false. If any one of those claims above is true then all of the rest would have to be false. But as it stands now, we do not have enough information about this subject to make a determination about whether any of them would even be hypothetically true or false. All of them must be considered undetermined without more information.

If the subject were more defined, such as Romulans, or Klingons, or Vulcans, then the claims would have a hypothetical truth value. 'All Romulans would be two-eyed creatures if there were Romulans' is true. It is not analytic, but it is known to be true based upon the fictional concept of Romulans. Here is yet another oddity about Venn's interpretation. Singular propositions about a specific subject are correctly treated as universal statements because if there is only one member of the category then either it has the predicate or it does not, and this would be applicable to the entire category (of one). So if we were to say 'Mr. Spock has two eyes' that is true for Venn's interpretation because it is universal. Actually, the claim would be considered true no matter which number of eyes you chose, which is a separate problem in its own right. That means that you could make any categorical claim about any specific fictional character that you want, and because they are always universal they would always be considered true, no matter what they say. Even something like 'Mr. Spock is female'. But we will ignore that little problem for the moment to focus on a different issue. The earlier claim, in categorical form, could be stated as 'All things identical to Mr. Spock are things that have exactly two eyes' and this would be considered true. But since a particular claim implies the actual existence of the subject, 'Some Vulcans have exactly two eyes' would be considered false. Now how could it be true that Mr. Spock, a Vulcan, has exactly two eyes, and at the same time false that at least one Vulcan has exactly two eyes? I am convinced that Spock himself would think that all of this was highly illogical.

We have considered at length what the truth value of a proposition would be when the subject category is empty. The next question is what its truth value would be if the predicate category is

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[9] One might argue that all the claims could be true at the same time because if it is true that an alien has 8 eyes then it would also be true that it has 4 eyes. The problem with this is that it does not work the other way around. We would not say that if it is true that a horse has 2 eyes then it must also be true that it has 8 eyes, and the argument does not work any better for aliens than for horses. Whenever a specific number is used it implies that there is not more or less than that number, or you would have said that number instead. But the predicate could easily be changed to say 'exactly 2 eyes' or 'two and two only' or 'two-eyed creature' to make it more explicit if necessary.

empty. The Venn diagram in those cases would have the entire predicate category shaded out. Venn believed that a universal claim did not necessarily imply that either category had members. However, there are some significant differences in how the subject and the predicate function in a categorical claim. The modern interpretation, taking its cues from Venn and Boole, only consider them to be two separate classes, and a categorical claim is only one of class inclusion or exclusion. This is the broadest possible interpretation of predication, but in many cases it is overly broad, and as a result, part of the intended meaning is lost.

Aristotle's original treatment of the predicate was as a property, characteristic, or attribute of the subject. In 'All x are white' the claim is not just that x has membership in both categories, but that whiteness is a property of x, and that all x have this property. Aristotle said that the predicate 'is in' or is 'said of' the subject. Being a mammal could be considered characteristic of, or a property that all dogs have. This is the underlying reason why a dog has membership in the 'mammals' category. The subject term in a categorical proposition always refers to some object, whether real or imagined, whereas the predicate term sometimes refers to objects, but often it is only an attribute or characteristic of things rather than an independent thing itself, as in the case of 'white' for the example above. You can have a class of white things, but 'white' itself is not an object, it is a property, and it does not make much sense to treat it as though it were an object. Because of this, if you tried using it as the subject of a proposition rather than the predicate, as in 'All white things are x' it would not make much sense. I cannot think of anything that x might stand for that would make that claim true except 'white', which of course would make it tautological, but essentially worthless.

In relation to English grammar, the subject usually refers to a noun, and the predicate is often a verb, or a phrase that includes a verb. In fact, in grammar, 'predicate' is defined as 'the part of a sentence or clause containing a verb and stating something about the subject'. Sometimes a predicate could sensibly be used as a noun/subject in another context, for example 'mammals' could easily be used as either subject or predicate, but this is not the case for many predicates. For example, in 'Jack ran' the subject is 'Jack', and the predicate is 'ran'. You could think of 'ran' as a separate class of objects, as in 'things that ran' but it only really makes sense to have this class function as a predicate. If we tried to use it as a subject we would end up with some very odd categorical propositions, such as 'Things that ran went to the store', or 'Running did his laundry' or 'Things that ran are happy'. These examples, and many more just like them, show that the subject and the predicate function differently in a categorical proposition, just as nouns and verbs function differently in a sentence. They are not interchangeable, and they do not merely represent classes of objects. The predicate is meant to describe or provide additional information about the subject. The claim that is being made in the proposition is about the subject, not the predicate; in fact, the predicate *is* the claim, and the truth or falsity of the proposition depends upon whether that claim is accurate.

A weakness even in Boole's mathematical interpretation is that with mathematical equations one can only capture the part of the claim concerning class inclusion or exclusion. Like any translation, something is lost, and here the loss is significant. Logic does not correspond perfectly to mathematics, though they do share some family resemblances. Many have tried in vain to reduce one to the other, but they are just not equivalent. You cannot reduce Chess to Checkers; one is simply a more complicated game, and there are some differences in the rules. We need to accept that while logic and math have some similarities, there are also some significant differences as well.

In some cases it may seem a little bit odd to consider a predicate as a property or attribute of the subject because we are used to thinking only of physical properties and attributes. But if we just think of it as a characteristic that all members of that category share, then it is not so odd. ‘All dogs are mammals’ could also be rendered as ‘All dogs share the common characteristic of being a mammal’ and/or ‘No dogs lack the characteristic of being a mammal’. Not all properties have to be physical properties. One must also remember that it would only be a property in reference to that particular claim; it could also, in some cases, be a separate category of objects.

I said previously that making claims about an empty subject category’s predicates is akin to division by zero, because the predicate would not apply to anything, and that all such claims are undefined. When the predicate category is empty it is more like having a zero in the numerator. Such claims do have a defined truth value. To return to the prior analogy, if a student does not complete any of the assignments for the class then they would have a zero out of however many points were possible, and this would result in a clear and definitive grade of 0, or F. Similarly, when the subject has members and the predicate category is empty obviously the subject must not have that predicate as one of its properties. Thus, the affirmative claims (A and I) would be false, and the negative ones (E and O) would be true. As an example, ‘All mammals are unicorns’ and ‘Some mammals are unicorns’ are clearly false, while ‘No mammals are unicorns’ and ‘Some mammals are not unicorns’ are clearly true.

However, as we did with the subject category, one could also consider the claim hypothetically when the predicate category is empty, as in ‘Some x would be y if there were y’. In the proposition ‘Some amulets have magical powers’ perhaps the predicate ‘things that have magical powers’ is an empty category because in fact there is no such thing. The claim would be false in actuality because it is asserting that there is at least one amulet that has a property or characteristic which in reality none of them have. There are real amulets, of course, so that makes this a false claim about an actual subject. But the claim could be rendered as a counterfactual, such as ‘Some amulets would have magical powers if there were such a thing as objects with magical powers’, which would be true. One could also stipulate that the claim is meant for a different context than the actual world, such as ‘In fantasy literature some amulets have magical powers’ or ‘According to the mythology and legends of many ancient cultures, some amulets were thought to have magical powers’. These would also be true because they are not committed to saying that things with magical powers really exist in the actual world.

In contraposition, ‘All S are P’ is considered logically equivalent to ‘All non-P are non-S’. But imagine a logically possible world in which nothing exists except cats. (There is not even anything for them to stand on, it is just cats floating in a void of empty space.) In this possible world, ‘Every cat is a mammal’ is true. It is also actual because the mammals category and the cats category are not empty. However, in this possible world, ‘Every non-mammal is a non-cat’ would not be true, it would be undefined, because the ‘non-mammals’ category is empty. Contraposition would only be hypothetically implied for this world, as in ‘Every non-mammal would be a non-cat, if there were non-mammals and non-cats’. Contraposition is valid in actuality only when all four categories referred to have at least one actual member.[10] Usually this would be the case. Admittedly, this example is a bit strange, but it proves the intended point. Here is another. If it is true that ‘All men are mortal’, and the category of ‘men’ has actual members, and ‘things that are mortal’ does as well, that does not necessarily mean that there are actual members of the ‘non-mortal’ or ‘immortal’ category (nor the ‘non-men’ category) simply because of contraposition. There could be, of course, but the logical relation alone does not prove that there is. Technically, all that is really implied by contraposition is ‘All immortals would be non-men if there were immortals and non-men’, though it would be unnecessary to state it in this way when the categories are known to have actual members. Since we know that the non-men category has members, it could also be ‘All immortals would be non-men if there are immortals’.

The same is true moving in the other direction as well. ‘All leprechauns are short’ is undefined because there are no actual leprechauns. However, the contrapositive, ‘All non-short things are non-leprechauns’ has an actual truth value because there are actual ‘non-short’ things, including giraffes, sky scrapers, people of average or above average height, etc. The claim is equivalent to ‘No things that are not short are leprechauns’ which is true, because there are actual members of the ‘non-short things’ category, and none of them are leprechauns. As these examples show, it cannot be inferred merely based upon logical relations alone which categories have members and which do not; that is an empirical question. Deductive reasoning cannot be used to go beyond the given information.

All that has been said so far could be applied to categorical syllogisms as well. Consider the following:

All aquatic things can swim
<u>All mermaids are aquatic</u>
Some mermaids can swim

Venn’s interpretation is that this syllogism (having the AAI-1 form) is not valid because of existential import. I would say that it is undetermined because the second premise and the conclusion are undefined. But the form is valid. It could be formulated as:

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[10] Venn discusses this issue on p. 137 and 138 of *Symbolic Logic*. He believes that if we say that universal propositions have existential import that commits us to saying not only that those categories have members, but also that other categories have members as well, such as non-S and non-P, because of logical relations such as contraposition, obversion, and conversion. My examples are intended to show why that is not the case.

All aquatic things can swim
<u>If there were mermaids, then all mermaids would be aquatic</u>
Thus, if there were mermaids, then at least some mermaids could swim

This syllogism is valid because the conclusion follows from the premises. It is also sound because the premises are true. But of course the condition 'If there were mermaids' has not been met in the actual world, so it is not applicable to, or in force in the actual world. The argument is hypothetical because one of the premises is only hypothetically true. Only a hypothetical conclusion may be drawn from a hypothetical premise.

Implications for Predicate Logic

I think much of the reason that people today are so tied to Venn's interpretation of categorical logic is because that interpretation has been carried over into predicate logic, which is so committed to distinguishing between universal and particular claims based upon existential import that an 'existential quantifier' is used to symbolize particular propositions. The symbol (∃x) means 'there exists an x such that'. A particular statement in predicate logic is formed by combining the existential quantifier with the dot operator, which stands for conjunction, or 'and'. The I statement is symbolized as: (∃x)(Sx • Px) and the O statement is: (∃x)(Sx • ~Px). What these literally stand for is 'There exists an x such that x is an S and x is a P' or 'Something exists that is both an S and a P', and 'There exists an x such that x is an S and x is not a P' or 'Something exists that is an S and not a P' respectively.

Universal statements in predicate logic are translated using the horseshoe symbol ⊃ which stands for a conditional statement. The symbol (x) is a variable that, just like for algebra, can stand for anything in the universe. As one would expect for a universal claim, the (x) symbol, or sometimes it is also symbolized as ∀x, means that the statement is making an assertion about every member of the S class. The A statement is formulated as: (x)(Sx ⊃ Px) and E is: (x)(Sx ⊃~Px). These mean, in ordinary language, 'For any x, if x is an S, then x is a P' or 'If anything is an S, then it is a P' and 'For any x, if x is an S, then x is not a P' or 'If anything is an S, then it is not a P' respectively.

As one can see, there is an almost perfect correspondence between Venn's interpretation of categorical logic, or the so-called 'Boolean interpretation' (though as I have argued, I do not think Boole was necessarily committed to this) and the predicate logic interpretation. But it would not have to be so. If one is convinced that Venn's interpretation is in error, changes can be made in predicate logic to bring it into harmony with what I have argued for above.

The first step would be to replace the existential quantifier with a particular quantifier. Since all of the claims have existential import there is no reason to focus so much upon existence for particular claims. All that a particular quantifier is meant to do is to show that the claim refers to part of the subject category rather than all of it. If the existential symbol is used at all it should be reserved only for the rare instances in which existence is specifically stated as part of the claim.

Another problem with the symbolization comes from using a conjunction to express predication. The idea is supposed to be that the conjunction represents an overlap of the two categories, and shows, in the case of affirmative claims, that there is something which is a member of both. For the mathematical interpretation of categorical logic a conjunctive statement works just fine because the claim is only one of class inclusion or exclusion. However, for some propositions, a significant part of the meaning is lost when this is all that is conveyed. Predication is more than just class inclusion or exclusion. To say that the predicate is an attribute or characteristic of the subject requires more than merely conjoining the two terms together. 'Some dogs are fast' does have an equivalent truth value to 'Some fast things are dogs' because one implies the other, but they do not have an equivalent meaning. In one, the claim is about dogs, and the claim being made about them is that at least one is fast; in the other, the claim is about fast things, and the claim is that at least one of those things is a dog. Those are not identical claims. But in a conjunction, the order of the simple statements does not matter: $D \bullet F$ is no different than $F \bullet D$. Thus, $(\exists x)(Dx \bullet Fx)$ and $(\exists x)(Fx \bullet Dx)$ would be saying exactly the same thing. That may be fine for the mathematical interpretation: 6 x 3 gives you the exact same answer as 3 x 6; if all that is being claimed is that there is something which is a member of both categories then either way of stating the claim performs that function equally well. But I do not think that it expresses the full meaning of predication very well. The good thing about a more Aristotelian sense of predication is that it can be used to express either class inclusion or exclusion, or the predicate could be considered a property or attribute of the subject, or it could be used in both senses.

I think the best way to represent this is through what I would call a limited conditional. Here is how I would symbolize the four categorical statements in predicate logic:

A: For every x, if x is S then x is P	$Ux(Sx \rightarrow Px)$
E: For every x, if x is S then x is not P	$Ux(Sx \rightarrow \sim Px)$
I: For some x, if x is S then x is P	$Px(Sx \rightarrow Px)$
O: For some x, if x is S then x is not P	$Px(Sx \rightarrow \sim Px)$

'Ux' stands for universal, of course, and 'Px' for particular. These designations are quite similar to how the quantifiers are used in categorical logic, being equivalent to 'all' and 'some' respectively.

The advantage of symbolizing the particular statements in this way is that it accurately shows that the order of the terms is important: 'If A then B' does not have the same meaning as 'If B then A'. It also expresses a stronger relationship between the terms. For the particulars, there is necessity that at least one member of the subject class has, or lacks the predicate. Obviously this necessity does not extend to all members of the category, but it is no longer just an arbitrary pairing of the terms either. This is a good way to show the predicate as a property or characteristic of that subject. However, it is also flexible enough to be used to express class inclusion and exclusion as well. For universals, being a member of the subject category is sufficient to guarantee inclusion in the predicate category in the case of A, or exclusion in the case of E. Membership in the predicate category is necessary for membership in the subject category for A, while not being a member of the predicate category is necessary for membership in the subject category for E. The same relationships hold for particulars, they just do not extend to all members of the category. For I, if x is one of the particular members of the subject category that the claim identifies, then this is sufficient to know that it has that predicate, or that it is also a member of the predicate class, while for O, if x is the particular member of S identified, then this is sufficient to guarantee that it does not have that predicate. Similarly, for I, if we know that x is not a member of P, then we would know that it could not be the particular S identified by the claim (though it could still be some other S). Similarly, for O, if we knew that x was in fact a P, then assuming that the claim is true, we would know that x could not be the particular member of S that the statement identifies, as not having membership in the P class is a necessary condition for being that particular member of S.

There is a rather elegant symmetry between the four statements when using these formulations. $\sim Ux(Sx \rightarrow \sim Px)$ and $Px(Sx \rightarrow Px)$ are logically equivalent. ‘It is not the case that for all x, if x is an S, then x is not P’ is equivalent to ‘For some x, if x is an S then x is a P’. As is $\sim Px(Sx \rightarrow \sim Px)$ and $Ux(Sx \rightarrow Px)$. ‘It is not the case that for some x, if x is an S, then x is not a P’ is equivalent to ‘For all x, if x is an S then x is a P’.

A: $Ux(Sx \rightarrow Px)$ negation of O: $\sim Px(Sx \rightarrow \sim Px)$	E: $Ux(Sx \rightarrow \sim Px)$ negation of I: $\sim Px(Sx \rightarrow Px)$
I: $Px(Sx \rightarrow Px)$ negation of E: $\sim Ux(Sx \rightarrow \sim Px)$	O: $Px(Sx \rightarrow \sim Px)$ negation of A: $\sim Ux(Sx \rightarrow Px)$

Counterfactuals could be used when the subject and/or predicate class does not have actual members. Perhaps the existential symbol could be useful here to show the counterfactual nature of the claim. ‘If there were mermaids, then every mermaid would be female’ would be symbolized as: $\exists x(Mx) \rightarrow Ux(Mx \rightarrow Fx)$, or ‘If there existed any x that were M, then all x that were M would be x that are F’. This is true. ‘If there were mermaids, then no mermaids would be female’ would be symbolized as $\exists x(Mx) \rightarrow Ux(Mx \rightarrow \sim Fx)$ and would obviously be false. ‘If there were mermaids, then some of them would have blonde hair’ would be symbolized as: $\exists x(Mx) \rightarrow Px(Mx \rightarrow Bx)$, or ‘If there existed any x that were M, then some x that were M would be x that are B’. This would quite likely be true. ‘If there were mermaids, then some of them would not have blonde hair’ would be $\exists x(Mx) \rightarrow Px(Mx \rightarrow \sim Bx)$ and would also very likely be true. However, because it is not the case that mermaids actually exist, the antecedent condition has not been met (at least not in the actual world) so all of the claims have only hypothetical rather than actual truth values.

These statements could also be symbolized without the existential quantifier. To say that mermaids exist, it could either be ∃x(Mx) or Px(Mx). The latter would mean 'For some x, x is M' or 'Some things are mermaids'. (You would not want to use 'Ux' or it would be saying that everything is a mermaid). To say that mermaids do not exist, or 'It is not the case that there are any mermaids', 'There are no mermaids', etc., it would be ~∃x(Mx) or ~Px(Mx). This could also be symbolized using a universal quantifier, which may fit better with the wording for some statements. It could be Ux[~(Mx)] meaning 'For any x, it is not the case that x is M', or Ux(~Mx), 'For any x, x is a non-M', which of course means 'Everything is a non-mermaid'.

Propositional Logic

Finally, similar modifications could also be made to propositional logic. An A statement in propositional logic is symbolized as: S→P, and E is: S→~P. All that is needed to represent the particulars in the same manner is to have some symbol indicate that the claim is meant to be particular. We could use *v* for this purpose if we wanted to stay consistent with a symbol that has been used to represent 'some' in the past. It may be a bit arbitrary, but almost any letter or symbol will do, though in order to avoid confusion it would be good to not use letters or symbols frequently used elsewhere, such as x, s, or p. The I statement could thus be expressed as: *v*(S→P), meaning 'In some instances, if S then P'. The O statement would of course be *v*(S→~P). One could, for the sake of greater consistency, choose some letter or symbol to represent universals as well, but this is not really necessary because it is already implied in their current forms that A means 'In all instances, if S then P' and E is 'In all instances, if S then not P'.

Some say that S→P does not necessarily imply that S exists, but I beg to differ. It is not really saying 'If there are any S then they are P' it is saying 'If S then P' which does assume that there are S (and P). To correctly represent the former claim it would have to be S→(S→P). This is how a counterfactual would be represented. If S or P did not really exist, a counterfactual could be symbolized as: S→(S→P) or P→(S→P). 'If there were unicorns, then some of them would be white' would be symbolized as: U→*v*(U→W) and would probably be true. These changes are not quite as seamless as those made in predicate logic, but I think it still expresses roughly the same thing.

2016